KEY SKILLS TRAINING LEVEL 3

mathsquad

-skill development-

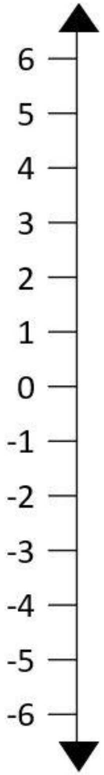
Contents

1 adding and subtracting integers

Questions Part 1 of 8 – Adding Positive Numbers Using a Number Line

1.1 Calculate the following additions using the number line on the right to assist your calculation.

a.	$^{-}4+3$	b.	$^{-}2+5$	c.	$^{-}2+7$	d.	$^{-}5+3$
e.	$^{-}2+2$	f.	$^{-}1+4$	g.	$^{-}4+6$	h.	$^{-}5+0$
i.	$^{-}2+6$	j.	$^{-}4+1$	k.	$^{-}3+6$	l.	$^{-}3+2$
m.	$^{-}2+3$	n.	$^{-}5+7$	o.	$^{-}3+5$	p.	$^{-}1+2$
q.	$^{-}4+4$	r.	$^{-}6+3$	s.	$^{-}6+1$	t.	$^{-}2+1$



Answers

a. -1 b. 3 c. 5 d. -2 e. 0 f. 3 g. 2 h. -5 i. -5 k. 4 j. -3 k. 3 l. -1
m. 1 n. 2 o. 2 p. 1 q. 0 r. 3 s. -5 t. -1

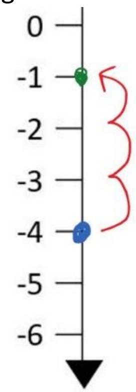
Helpful Information

Strategy – Adding and Subtracting Positive Numbers using a Number Line

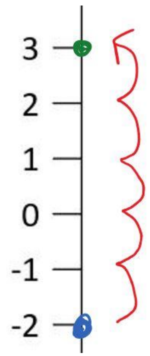
1. Put your finger (or pen tip) on the number line at the position of the first number
2. Then
   a. If adding a positive number move in the positive direction ↑
   b. If subtracting a positive number move in the negative direction ↓

Examples

Question: Calculate $^{-}4+3$ using a number line on the right to assist your calculation.
Thought process: Using the above strategy we…
1. Start at -4
2. Move 3 in the positive direction ↑
Answer: -1



Question: Calculate $^{-}2+5$ using a number line on the right to assist your calculation.
Thought process: Using the above strategy we…
1. Start at -2
2. Move 5 in the positive direction ↑
Answer: 3



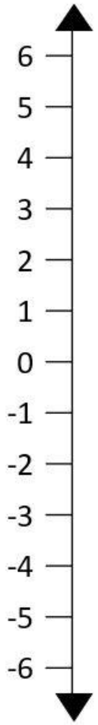
Questions Part 2 of 8 – Subtracting Positive Numbers Using a Number Line

1.2 Calculate the following subtractions using the number line on the right to assist your calculation.

a. $^{-}3-2$	b. $2-3$	c. $4-6$	d. $^{-}1-3$
e. $^{-}1-4$	f. $^{-}2-2$	g. $^{-}1-3$	h. $6-6$
i. $0-5$	j. $^{-}1-4$	k. $6-2$	l. $1-4$
m. $3-3$	n. $1-3$	o. $^{-}2-3$	p. $3-2$
q. $^{-}5-1$	r. $^{-}3-3$	s. $2-1$	t. $^{-}4-1$

Answers

a. -5 b. -1 c. -2 d. -4 e. -5 f. -5 g. -4 h. 0 i. -5 j. -5 k. 4 l. -3 m. 0 n. -2 o. -5 p. 1 q. -6 r. -6 s. 1 t. -5



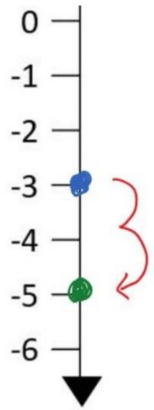
Helpful Information

Strategy – Adding and Subtracting Positive Numbers using a Number Line

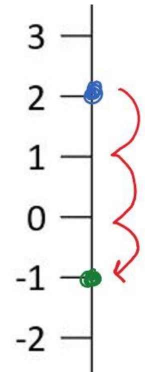
1. Put your finger (or pen tip) on the number line at the position of the first number
2. Then
   a. If adding a positive number move in the positive direction ↑
   b. If subtracting a positive number move in the negative direction ↓

Examples

Question: Calculate $^{-}3-2$ using a number line on the right to assist your calculation.
Thought process: Using the above strategy we…
1. Start at -3
2. Move 2 in the negative direction ↓
Answer: -5



Question: Calculate $2-3$ using a number line on the right to assist your calculation.
Thought process: Using the above strategy we…
1. Start at 2
2. Move 3 in the negative direction ↓
Answer: -1



Questions Part 3 of 8 – Adding and Subtracting Positive Numbers Using a Number Line

1.3 Calculate the following using a number line to assist your calculation if you wish.

a. $5-1$	b. $^{-}2-1$	c. $^{-}3+2$	d. $^{-}2+3$	e. $^{-}2-2$
f. $^{-}5+2$	g. $^{-}4-1$	h. $^{-}6+2$	i. $^{-}2+5$	j. $^{-}1+0$
k. $3-3$	l. $3-1$	m. $^{-}2-4$	n. $^{-}1-3$	o. $6-1$
p. $^{-}4+3$	q. $^{-}6+4$	r. $^{-}5+2$	s. $1-2$	t. $^{-}1+1$

Answers

a. 4 b. -3 c. -1 d. 1 e. 0 f. -3 g. -5 h. -4 i. 3 j. -1 k. 0 l. 2 m. -6 n. -4 o. 5 p. -1 q. -2 r. -3 s. -1 t. 0

Helpful Information

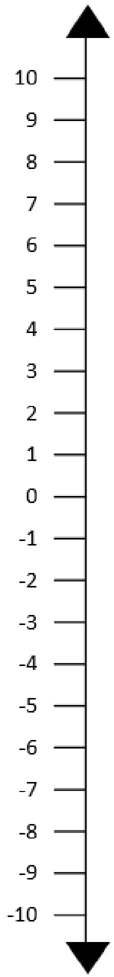
Strategy – Adding and Subtracting Positive Numbers using a Number Line

1. Put your finger (or pen tip) on the number line at the position of the first number
2. Then
   a. If adding a positive number move in the positive direction ↑
   b. If subtracting a positive number move in the negative direction ↓

Questions Part 4 of 8 – Adding negative integers

1.4 Calculate the following additions using the number line on the right to assist your calculation.

a. $2 + {}^{-}5$	b. ${}^{-}3 + {}^{-}2$	c. $1 + {}^{-}3$	d. $4 + {}^{-}3$
e. $0 + {}^{-}5$	f. $7 + {}^{-}3$	g. ${}^{-}4 + {}^{-}5$	h. ${}^{-}5 + {}^{-}1$
i. $5 + {}^{-}3$	j. ${}^{-}2 + {}^{-}6$	k. $5 + {}^{-}5$	l. ${}^{-}6 + {}^{-}5$
m. $1 + {}^{-}2$	n. $0 + {}^{-}2$	o. $5 + {}^{-}6$	p. ${}^{-}4 + {}^{-}2$
q. ${}^{-}5 + {}^{-}4$	r. ${}^{-}6 + {}^{-}1$	s. $2 + {}^{-}6$	t. $0 + {}^{-}6$



Answers

a. -3 b. -5 c. -2 d. 1 e. -5 f. 4 g. -9 h. -6 i. 2 j. -8 k. 0 l. -11 m. -1 n. -2 o. -1 p. -6 q. -9 r. -7 s. -4 t. -6

Helpful Information

Strategy – Adding and Subtracting Negative Numbers using a Number Line

1. Put your finger (or pen tip) on the number line at the position of the first number
2. Then
   a. If adding a negative number move in the negative direction ↓
   b. If subtracting a negative number move in the positive direction ↑

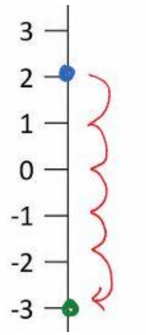
Examples

Question: Calculate $2 + {}^{-}5$ using a number line on the right to assist your calculation.

Thought process: Using the above strategy we…

1. Start at 2
2. Move 5 in the negative direction ↓

Answer: -3

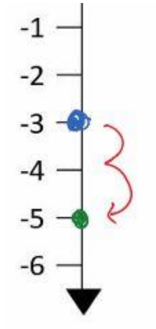


Question: Calculate ${}^{-}3 + {}^{-}2$ using a number line on the right to assist your calculation.

Thought process Using the above strategy we…

1. Start at -3
2. Move 2 in the negative direction ↓

Answer: -5



Questions Part 5 of 8 – Subtracting negative integers

1.5 Calculate the following subtractions using the number line on the right to assist your calculation.

a. $^{-}5 - {}^{-}2$ b. $2 - {}^{-}4$ c. $0 - {}^{-}4$ d. $^{-}3 - {}^{-}1$

e. $^{-}3 - {}^{-}5$ f. $^{-}5 - {}^{-}5$ g. $^{-}6 - {}^{-}1$ h. $4 - {}^{-}3$

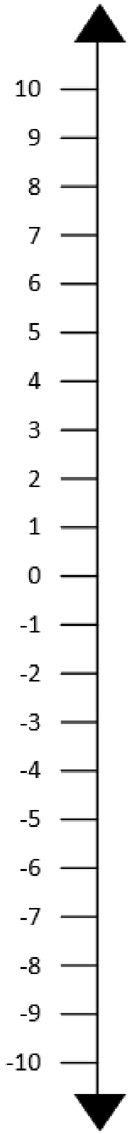
i. $^{-}6 - {}^{-}4$ j. $^{-}1 - {}^{-}4$ k. $^{-}6 - {}^{-}5$ l. $2 - {}^{-}5$

m. $6 - {}^{-}1$ n. $^{-}5 - {}^{-}3$ o. $^{-}2 - {}^{-}3$ p. $1 - {}^{-}4$

q. $4 - {}^{-}1$ r. $^{-}4 - {}^{-}1$ s. $^{-}4 - {}^{-}4$ t. $^{-}2 - {}^{-}1$

Answers

a. -3 b. 6 c. 4 d. -2 e. 2 f. 0 g. -5 h. 7 i. -2 j. 3 k. -1 l. 7 m. 7 n. -2 o. 1 p. 5 q. 5 r. -3 s. 0 t. -1



Helpful Information

Strategy – Adding and Subtracting Negative Numbers using a Number Line

1. Put your finger (or pen tip) on the number line at the position of the first number
2. Then
   a. If adding a negative number move in the negative direction ↓
   b. If subtracting a negative number move in the positive direction ↑

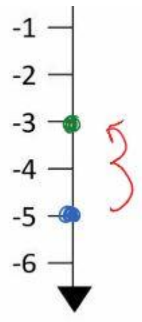
Examples

Question: Calculate $^{-}5 - {}^{-}2$ using a number line on the right to assist your calculation.

Thought process: Using the above strategy we...

1. Start at -5
2. Move 2 in the positive direction ↑

Answer: -3

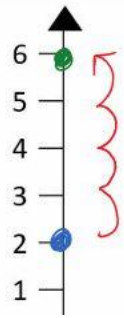


Question: Calculate $2 - {}^{-}4$ using a number line on the right to assist your calculation.

Thought process: Using the above strategy we...

1. Start at 2
2. Move 4 in the positive direction ↑

Answer: 6



Questions Part 6 of 8 – Adding and subtracting negative integers

1.6 Calculate the following using a number line to assist your calculation if you wish.

a. $^{-}5 + {}^{-}2$	b. $^{-}3 + {}^{-}1$	c. $^{-}4 - {}^{-}4$	d. $^{-}6 - {}^{-}5$
e. $^{-}1 - {}^{-}2$	f. $^{-}6 + {}^{-}2$	g. $4 - {}^{-}1$	h. $^{-}2 + {}^{-}3$
i. $4 + {}^{-}9$	j. $3 + {}^{-}7$	k. $^{-}3 + {}^{-}3$	l. $1 - {}^{-}3$
m. $^{-}5 - {}^{-}4$	n. $2 - {}^{-}4$	o. $^{-}1 + {}^{-}3$	p. $^{-}4 + {}^{-}5$
q. $1 + {}^{-}7$	r. $^{-}1 - {}^{-}4$	s. $^{-}1 + {}^{-}3$	t. $0 - {}^{-}2$

Answers

a. -7 b. -4 c. 0 d. -1 e. 1 f. -8 g. 5 h. -5 i. -5 j. -4 k. -6 l. 4 m. -1 n. 6 o. -4 p. -9 q. -6 r. 3 s. -4 t. 2

Helpful Information

Strategy – Adding and Subtracting Negative Numbers using a Number Line

1. Put your finger (or pen tip) on the number line at the position of the first number

Questions Part 7 of 8– Adding and subtracting integers

1.7 Calculate the following.

a. $5 + {}^{-}1$	b. $^{-}2 - 1$	c. $^{-}3 - {}^{-}2$	d. $^{-}2 + 3$
e. $^{-}2 + {}^{-}2$	f. $^{-}5 - {}^{-}2$	g. $^{-}4 - 1$	h. $^{-}6 + {}^{-}2$
i. $^{-}2 - {}^{-}5$	j. $^{-}1 + 0$	k. $3 - 3$	l. $3 + {}^{-}1$
m. $^{-}2 - 4$	n. $^{-}4 - 3$	o. $6 + {}^{-}1$	p. $^{-}4 + {}^{-}3$
q. $^{-}7 - {}^{-}5$	r. $^{-}5 + {}^{-}2$	s. $1 - 2$	t. $^{-}1 + 1$

Answers

a. 4 b. -3 c. -1 d. 1 e. -4 f. -3 g. -5 h. -8 i. 3 j. -1 k. 0 l. 2 m. -6 n. -7 o. 5 p. -7 q. -2 r. -7 s. -1 t. 0

Helpful Information

Strategy – Adding and Subtracting Integers using a Number Line

The song below is helpful for remembering how to add and subtract integers using a number line.

♫Addition and subtraction are really big words so need to be handled with care
Get your number line out, look at the first number, and put your finger there

The only thing to know is which way to go, it's easy so don't you frown
To add positive or take negative go up if not go down ♫

Questions Part 8 of 8 – Showing a pattern that helps us see why subtracting an integer requires a move in the positive direction

1.8 Answer the following questions.

Part 1	**Part 2**
a. $3 - 3$ b. $3 - 2$ c. $3 - 1$ d. $3 - 0$ e. $3 - {}^{-}1$ f. $3 - {}^{-}2$	Explain what happens to the answers as we subtract smaller numbers

Answers

Part 1 a. 0 b. 1 c. 2 d. 3 e. 4 f. 5

Part 2 As we subtract smaller numbers the answers becomes bigger.

2 multiplying and dividing integers

Questions Part 1 of 2 – Multiplying Integers

2.1 Calculate the following.

a. $6 \times {}^{-}2$	b. ${}^{-}1 \times {}^{-}4$	c. ${}^{-}3 \times 6$	d. ${}^{-}7 \times {}^{-}3$	e. $6 \times {}^{-}2$
f. ${}^{-}3 \times {}^{-}2$	g. ${}^{-}5 \times {}^{-}7$	h. ${}^{-}3 \times {}^{-}5$	i. $2 \times {}^{-}1$	j. ${}^{-}4 \times 4$
k. ${}^{-}2 \times 2$	l. ${}^{-}1 \times {}^{-}3$	m. $6 \times {}^{-}4$	n. ${}^{-}5 \times 0$	o. ${}^{-}1 \times {}^{-}4$
p. ${}^{-}3 \times {}^{-}1$	q. ${}^{-}2 \times 7$	r. ${}^{-}6 \times {}^{-}4$	s. ${}^{-}5 \times 3$	t. ${}^{-}1 \times {}^{-}6$

Answers

a. -12 b. 4 c. -18 d. 21 e. -12 f. 6 g. 35 h. 15 i. -2 j. -16 k. -4 l. 3 m.-24 n. 0 o. 4 p. 3 q. -14 r. 24 s. -15 t. 6

Helpful Information

Strategy – Multiplying Two Integers

1. Multiply the two values ignoring the signs
2. Then
   a. If the signs were the <u>same</u> make the answer <u>positive</u>
   b. the signs were <u>different</u> make the answer <u>negative</u>

Examples

Question: $6 \times {}^{-}2$

Thought Process: Multiplying 6 by 2 gives 12. 6 and -2 have different signs so the answer will be negative. The answer is -12.

Answer: -12

Question: ${}^{-}1 \times {}^{-}4$

Thought Process: Multiplying 1 by 4 gives 4. -1 and -4 have the same sign so the answer will be positive. The answer is 4.

Answer: 4

Questions Part 2 of 2 – Dividing Integers

2.2 Calculate the following.

a. ${}^{-}35 \div {}^{-}5$	b. ${}^{-}12 \div 3$	c. $0 \div {}^{-}4$	d. ${}^{-}5 \div {}^{-}5$
e. ${}^{-}1 \div {}^{-}1$	f. ${}^{-}32 \div 4$	g. ${}^{-}30 \div {}^{-}6$	h. ${}^{-}20 \div 4$
i. $60 \div {}^{-}12$	j. ${}^{-}16 \div 4$	k. ${}^{-}18 \div 3$	l. ${}^{-}12 \div {}^{-}3$
m. ${}^{-}5 \div 5$	n. ${}^{-}24 \div {}^{-}4$	o. ${}^{-}36 \div 9$	p. ${}^{-}45 \div 5$
q. $42 \div {}^{-}7$	r. ${}^{-}12 \div {}^{-}4$	s. ${}^{-}20 \div 5$	t. ${}^{-}4 \div {}^{-}2$

Answers

a. 7 b. -4 c. 0 d. 1 e. 1 f. -8 g. 5 h. -5 i. -5 j. -4 k. -6 l. 4 m. -1 n. 6 o. -4 p. -9 q. -6 r. 3 s. -4 t. 2

Helpful Information

Strategy – Dividing Two Integers

1. Divide the two values ignoring the signs
2. Then
   a. If the signs were the <u>same</u> make the answer <u>positive</u>
   b. the signs were <u>different</u> make the answer <u>negative</u>

Examples

Question: ${}^{-}35 \div {}^{-}5$

Thought Process: Dividing 35 by 5 gives 7. -35 and -5 have the same sign so the answer will be positive. The answer is 7.

Answer: 4

Question: ${}^{-}12 \div 3$

Thought Process: Dividing 12 by 3 gives 4. -12 and 3 have different signs so the answer will be negative. The answer is -4.

Answer: -12

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3 index laws

Questions Part 1 of 4 – Multiplying with the same base

3.1 Simplify the following giving your answer in index form where appropriate.

a. $5^3 \times 5^6$	b. 4×4^7	c. $3^6 \times 3^2$	d. $9^4 \times 9$
e. $3^7 \times 3^4$	f. 3×3^2	g. $6^2 \times 6^2$	h. $2^5 \times 2^3$
i. $3^2 \times 3^7$	j. $9^3 \times 9^3$	k. $8^6 \times 8^6$	l. $6^5 \times 6^3$
m. $4^7 \times 4^5$	n. $4^4 \times 4$	o. $2^2 \times 2^6$	p. $3^7 \times 3$
q. 9×9^5	r. $7^7 \times 7^2$	s. $9^6 \times 9^5$	t. $5^2 \times 5^6$

Answers

a. 5^9 b. 4^8 c. 3^8 d. 9^5 e. 3^{11} f. 3^3 g. 6^4 h. 2^8 i. 3^9 j. 9^6 k. 8^{12} l. 6^8 m. 4^{12} n. 4^5 o. 2^8 p. 3^8 q. 9^6 r. 7^9 s. 9^{11} t. 5^8

Helpful Information

A number is said to be in **index form** if it involves a power. For example, 2^3, 3^{-2} and -4^2 are all in index form, where 8 and -16 are not in index form.

When a number is written in index form, we can call the power the index and the number raised to that power is called the base. For example, 3^4 has an index of 4 and a base of 3.

When two numbers in index form have the same base are multiplied the expression can be simplified by adding the powers. For example,

$$3^2 \times 3^5 = 3^{2+5} = 3^7$$

In general

$$\boxed{a^x \times a^y = a^{x+y}}$$

Note also that $a = a^1$ and so if no power is given it is helpful to re-write the expression with a power of 1 so you have "a power to add".

Examples

Question: Simplify $5^3 \times 5^6$ giving your answer in index form if appropriate.
Thought process: The two bases in the expression are equal so we can simplify by adding the powers.

$$5^3 \times 5^6 = 5^{3+6} = 5^9$$

Answer: 5^9

Question: Simplify 4×4^7 giving your answer in index form if appropriate.
Thought process: We begin by re-writing the expression as $4^1 \times 4^7$. The two bases in the expression are equal so we can simplify by adding the powers.

$$4^1 \times 4^7 = 4^{1+7} = 4^8$$

Answer: 4^8

Questions Part 2 of 4 – Dividing with the same base

3.2 Simplify the following giving your answer in index form where appropriate.

a.	$4^6 \div 4$	b.	$\frac{9^5}{9^5}$	c.	$7^8 \div 7^7$	d.	$\frac{6^6}{6^4}$
e.	$\frac{2^3}{2}$	f.	$9^7 \div 9^3$	g.	$8^6 \div 8^6$	h.	$\frac{4^5}{4^3}$
i.	$4^8 \div 4^7$	j.	$4^8 \div 4^6$	k.	$\frac{3^2}{3}$	l.	$\frac{8^6}{8^4}$
m.	$4^8 \div 4^2$	n.	$\frac{4^3}{4^2}$	o.	$9^8 \div 9^7$	p.	$7^4 \div 7^3$
q.	$\frac{5^3}{5^2}$	r.	$\frac{3^6}{3}$	s.	$8^8 \div 8^5$	t.	$\frac{5^5}{5^5}$

Answers

a. 4^5 b. 1 c. 7 d. 6^2 e. 2^2 f. 9^4 g. 1 h. 4^2 i. 4 j. 4^2 k. 3 l. 8^2 m. 4^6 n. 4 o. 9 p. 7 q. 5 r. 3^5 s. 8^3 t. 1

Helpful Information

When two numbers in index form have the same base are divided the expression can be simplified by subtracting the powers. For example,

$$7^5 \div 7^2 = 7^{5-2} = 7^3 \quad \text{and} \quad \frac{3^9}{3^2} = 3^{9-2} = 3^7$$

In general

$$\boxed{a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}}$$

Note also that $a^0 = 1$ and $a^1 = a$ and if an answer has a power of 0 or 1 then you must further simplify using these rules.

Examples

Question: Simplify $4^6 \div 4$ giving your answer in index form if appropriate.
Thought process: We begin by re-writing the expression as $4^6 \div 4^1$. The two bases in the expression are equal so we can simplify by subtracting the powers.

$$4^6 \div 4^1 = 4^{6-1} = 4^5$$

Answer: 4^5

Question: Simplify $\frac{9^5}{9^5}$ giving your answer in index form if appropriate.
Thought process: The two bases in the expression are equal so we can simplify by subtracting the powers.

$$\frac{9^5}{9^5} = 9^{5-5} = 9^0 = 1$$

Answer: 1

Question: Simplify $7^8 \div 7^7$ giving your answer in index form if appropriate.
Thought process: The two bases in the expression are equal so we can simplify by subtracting the powers.

$$7^8 \div 7^7 = 7^{8-7} = 7^1 = 7$$

Answer: 7

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Questions Part 3 of 4 – Powers of powers

3.3 Simplify the following giving your answer in index form where appropriate.

a.	$(5^4)^7$	b.	$(9^4)^0$	c.	$(2^6)^5$	d.	$(6^6)^4$
e.	$(5^2)^7$	f.	$(4^2)^3$	g.	$(4^8)^4$	h.	$(7^2)^7$
i.	$(7^8)^0$	j.	$(3^0)^6$	k.	$(2^3)^6$	l.	$(9^2)^7$
m.	$(3^3)^7$	n.	$(9^8)^7$	o.	$(8^8)^3$	p.	$(3^3)^5$
q.	$(8^2)^4$	r.	$(5^4)^4$	s.	$(5^0)^4$	t.	$(3^7)^2$

Answers

a. 5^{28} b. 1 c. 2^{30} d. 6^{24} e. 5^{14} f. 4^6 g. 4^{32} h. 7^{14} i. 1 j. 1 k. 2^{18} l. 9^{14} m. 3^{21} n. 9^{56} o. 8^{24} p. 3^{15} q. 8^8 r. 5^{16} s. 1 t. 3^{14}

Helpful Information

When a number in index form is further raised to a power the expression can be simplified by multiplying the powers. For example,

$$(3^5)^2 = 3^{5\times 2} = 3^{10}$$

In general

$$\boxed{(a^x)^y = a^{xy}}$$

Recall that $a^0 = 1$ and $a^1 = a$

Examples

Question: Simplify $(5^4)^7$ giving your answer in index form if appropriate.

Thought process: A power is raised to another power so can be simplified by multiplying the powers.

$$(5^4)^7 = 5^{4\times 7} = 5^{28}$$

Answer: 5^{28}

Question: Simplify $(9^4)^0$ giving your answer in index form if appropriate.

Thought process: A power is raised to another power so can be simplified by multiplying the powers.

$$(9^4)^0 = 9^{4\times 0} = 9^0 = 1$$

Answer: 1

Questions Part 4 of 4 – Simplifying using index laws

3.4 Simplify the following giving your answer in index form where appropriate.

a.	$7^6 \times 7^2$	b.	$6^7 \div 6^3$	c.	$\frac{5^6}{5}$	d.	$(2^3)^5$
e.	$4^8 \div 4^3$	f.	$(4^3)^0$	g.	$2^4 \times 2^2$	h.	$\frac{4^7}{4^6}$
i.	$\frac{6^4}{6}$	j.	$5^4 \times 5^5$	k.	$(3^2)^6$	l.	$9^2 \times 9^5$
m.	$6^3 \div 6^2$	n.	$\frac{3^6}{3^3}$	o.	$4^7 \times 4$	p.	$\frac{8^6}{8^2}$
q.	$3^5 \times 3^2$	r.	$(9^4)^7$	s.	$\frac{6^5}{6^2}$	t.	$(6^3)^3$

Answers

a. 7^8 b. 6^4 c. 5^5 d. 2^{15} e. 4^5 f. 1 g. 2^6 h. 4 i. 6^3 j. 5^9 k. 3^{12} l. 9^7 m. 6 n. 3^3 o. 4^8 p. 8^4 q. 3^7 r. 9^{28} s. 6^3 t. 6^9

Helpful Information

Index Laws	**Powers of One and Zero**
• $a^x \times a^y = a^{x+y}$ • $a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}$ • $(a^x)^y = a^{xy}$	• $a^1 = a$ • $a^0 = 1$

4 ratios

Questions Part 1 of 2 – Simplifying ratios

4.1 Simplify the following ratios.

a.	48:54	b.	36:27	c.	36:84	d.	9:6
e.	54:99	f.	55:40	g.	84:77	h.	42:48
i.	36:32	j.	99:63	k.	55:44	l.	18:22
m.	70:40	n.	96:132	o.	4:20	p.	121:77
q.	24:21	r.	22:18	s.	22:18	t.	18:6

Answers

a. 8:9 b. 4:3 c. 3:7 d. 3:2 e. 6:11 f. 11:8 g. 12:11 h. 7:8 i. 9:8 j. 11:7 k. 5:4 l. 9:11 m. 7:4 n. 8:11 o. 1:5 p. 11:7 q. 8:7 r. 11:9 s. 11:9 t. 3:1

Helpful Information

A **ratio** specifies the multiplicative relationship between 2 or more quantities. For example, if a group of students has 12 girls and 17 boys then the ratio of girls to boys is 12:17.

A **simplified ratio** is a ratio where the numbers within it are whole numbers and have no common factors. The ratios 2:3 and 4:15 are simplified where the ratios 6:9, $\frac{1}{2}:1$ and 0.4:1.5 are not simplified.

Equivalent ratios are formed by multiplying or dividing each number in the ratio by the same value. For example, $3:5 = 18:30 = 9:15$

Examples

Question: Simplify 48:54

Thought process: 2 is a common factor of 48 and 54 so

$$48:54 = 24:27$$

3 is a common factor of 24 and 27

$$24:27 = 8:9$$

8 and 9 have no common factors and so is a simplified ratio.

Answer: 8:9

Questions Part 2 of 2 – Sharing a quantity in a given ratio

4.2 Divide each whole number into the given ratio.

a.	35, 4:1	b.	21, 2:5	c.	36, 3:1	d.	49, 4:3	e.	90, 7:3
f.	20, 4:1	g.	15, 2:1	h.	50, 4:1	i.	100, 7:3	j.	24, 2:1
k.	30, 2:3	l.	30, 5:1	m.	28, 5:2	n.	21, 2:1	o.	12, 3:1
p.	36, 7:2	q.	72, 7:2	r.	35, 2:3	s.	45, 4:1	t.	72, 3:5

Answers

a. 28 and 7 b. 6 and 15 c. 27 and 9 d. 28 and 21 e. 63 and 27 f. 16 and 4 g. 10 and 5 h. 40 and 10 i. 70 and 30 j. 16 and 8 k. 12 and 18 l. 25 and 5 m. 20 and 8 n. 14 and 7 o. 9 and 3 p. 28 and 8 q. 56 and 16 r. 14 and 21 s. 36 and 9 t. 27 and 45

Examples

Question: Divide 35 in the ratio 4:1

Thought process: There are 4+1=5 parts to the ratio 4:1. 35÷5=7 so each part is worth 7. 4 parts of 7 is 28 and 1 part of 7 is 7, and so splitting 35 in the ratio 4:1 gives 28 and 7. (A quick check to make sure 28+7=35 is helpful!)

7	7	7	7	7

Answer: 28 and 7

Question: Divide 21 in the ratio 2:5

Thought process: There are 2+5=7 parts to the ratio 2:5. 21÷7=3 so each part is worth 3. 2 parts of 3 is 6 and 5 parts of 3 is 15, and so splitting 21 in the ratio 2:5 gives 6 and 15. (A quick check to make sure 6+15=21 is helpful!)

3	3	3	3	3	3	3

Answer: 6 and 15

5 Fraction arithmetic involving integers

Questions Part 1 of 2 – Multiplying a fraction by an integer

5.1 Calculate the following. Answers can be given as improper fractions or mixed numbers.

a. $-5\left(\frac{2}{3}\right)$	b. $-7\left(\frac{3}{7}\right)$	c. $-3\left(\frac{9}{10}\right)$	d. $-9\left(\frac{8}{9}\right)$
e. $-7\left(\frac{3}{8}\right)$	f. $-6\left(\frac{1}{11}\right)$	g. $-4\left(\frac{4}{7}\right)$	h. $2\left(\frac{3}{5}\right)$
i. $2\left(\frac{1}{2}\right)$	j. $-5\left(\frac{6}{7}\right)$	k. $-5\left(\frac{3}{10}\right)$	l. $-5\left(\frac{3}{4}\right)$
m. $-2\left(\frac{1}{2}\right)$	n. $-2\left(\frac{4}{9}\right)$	o. $8\left(\frac{1}{3}\right)$	p. $-9\left(\frac{2}{3}\right)$
q. $-6\left(\frac{1}{2}\right)$	r. $-7\left(\frac{7}{10}\right)$	s. $-5\left(\frac{2}{3}\right)$	t. $-4\left(\frac{5}{12}\right)$

Answers

a. $-\frac{10}{3}$ or $-3\frac{1}{3}$ b. -3 c. $-\frac{27}{10}$ or $-2\frac{7}{10}$ d. -8 e. $-\frac{21}{8}$ or $-2\frac{5}{8}$ f. $-\frac{6}{11}$
g. $-\frac{16}{7}$ or $-2\frac{2}{7}$ h. $\frac{6}{5}$ or $1\frac{1}{5}$ i. 1 j. $-\frac{30}{7}$ or $-4\frac{2}{7}$ k. $-\frac{3}{2}$ or $-1\frac{1}{2}$ l. $-\frac{15}{4}$ or $-3\frac{3}{4}$ m. -1
n. $-\frac{8}{9}$ o. $\frac{8}{3}$ or $2\frac{2}{3}$ p. -6 q. -3 r. $-\frac{49}{10}$ or $-4\frac{9}{10}$ s. $-\frac{10}{3}$ or $-3\frac{1}{3}$ t. $-\frac{5}{3}$ or $-1\frac{2}{3}$

Helpful Information

When a question requires us to combine fractions and whole numbers in some way it is often easiest to write the whole number as an improper fraction and then use standard fraction arithmetic.

Recall that $a = \frac{a}{1}$ for all whole numbers.

Strategy for Completing Arithmetic Involving Fractions and Whole Numbers

1. Convert all whole numbers to improper fractions
2. Use the standard approaches used in skills C06 and C07.

Examples

Question: $-5\left(\frac{2}{3}\right)$

Thought process: When two numbers are written next to each other with no operation in between a multiplication is implied so we have…

$$-5\left(\frac{2}{3}\right) = -5 \times \frac{2}{3} = -\frac{5}{1} \times \frac{2}{3} = -\frac{10}{3}$$

The answer can be left as $-\frac{10}{3}$ or written as a mixed number $-3\frac{1}{3}$

Answer: $-\frac{10}{3}$ or $-3\frac{1}{3}$

Question: $-7\left(\frac{3}{7}\right)$

Thought process: When two numbers are written next to each other with no operation in between a multiplication is implied so we have…

$$-7\left(\frac{3}{7}\right) = -7 \times \frac{3}{7} = -\frac{7}{1} \times \frac{3}{7} = -\frac{21}{7} = -3$$

Answer: -3

Questions Part 2 of 2 – Adding or subtracting a fraction and an integer

5.2 Calculate the following. Answers can be given as improper fractions or mixed numbers.

a. $-3+\frac{5}{2}$	b. $\frac{10}{7}-2$	c. $-5-\frac{5}{2}$	d. $-5+\frac{14}{11}$
e. $-4-\frac{5}{4}$	f. $\frac{7}{4}-3$	g. $-3+\frac{20}{11}$	h. $\frac{5}{3}-3$
i. $-3+\frac{5}{3}$	j. $5-\frac{14}{11}$	k. $5-\frac{23}{12}$	l. $\frac{3}{2}-5$
m. $-4-\frac{3}{2}$	n. $-3+\frac{19}{10}$	o. $-3+\frac{5}{4}$	p. $\frac{9}{8}-2$
q. $\frac{12}{7}-2$	r. $-5-\frac{3}{2}$	s. $\frac{16}{9}-5$	t. $\frac{8}{7}-3$

Answers

a. $-\frac{1}{2}$ b. $-\frac{4}{7}$ c. $-\frac{15}{2}$ or $-7\frac{1}{2}$ d. $-\frac{41}{11}$ or $-3\frac{8}{11}$ e. $-\frac{21}{4}$ or $-5\frac{1}{4}$ f. $-\frac{5}{4}$ or $-1\frac{1}{4}$ g. $-\frac{13}{11}$ or $-1\frac{2}{11}$ h. $-\frac{4}{3}$ or $-1\frac{1}{3}$ i. $-\frac{4}{3}$ or $-1\frac{1}{3}$ j. $\frac{41}{11}$ or $3\frac{8}{11}$ k. $\frac{37}{12}$ or $3\frac{1}{12}$
l. $-\frac{7}{2}$ or $-3\frac{1}{2}$ m. $-\frac{11}{2}$ or $-5\frac{1}{2}$ n. $-\frac{11}{10}$ or $-1\frac{1}{10}$ o. $-\frac{7}{4}$ or $-1\frac{3}{4}$ p. $-\frac{7}{8}$ q. $-\frac{2}{7}$
r. $-\frac{13}{2}$ or $-6\frac{1}{2}$ s. $-\frac{29}{9}$ or $-3\frac{2}{9}$ t. $-\frac{13}{7}$ or $-1\frac{6}{7}$

Helpful Information

When a question requires us to combine fractions and whole numbers in some way it is often easiest to write the whole number as an improper fraction and then use standard fraction arithmetic.

Recall that $a=\frac{a}{1}$ for all whole numbers.

Strategy for Completing Arithmetic Involving Fractions and Whole Numbers

1. Convert all whole numbers to improper fractions
2. Use the standard approaches used in skills C06 and C07.

Example

Question: Calculate $-3+\frac{5}{2}$

Thought process: Using the strategy above…

$$-3+\frac{5}{2}=-\frac{3}{1}+\frac{5}{2}=-\frac{6}{2}+\frac{5}{2}=-\frac{1}{2}$$

Answer: $-\frac{1}{2}$

6 converting fractions to recurring and terminating decimals

6.1 Express each of the fractions below as a decimal. Give answers to 4 decimal places where appropriate. State whether the decimal is recurring (R) or terminating (T).

a. $\frac{10}{11}$	b. $\frac{3}{5}$	c. $1\frac{3}{11}$	d. $\frac{3}{7}$	e. $1\frac{2}{5}$
f. $1\frac{1}{2}$	g. $1\frac{3}{10}$	h. $1\frac{1}{3}$	i. $\frac{1}{7}$	j. $\frac{5}{7}$
k. $\frac{7}{10}$	l. $1\frac{3}{5}$	m. $\frac{2}{11}$	n. $\frac{2}{7}$	o. $1\frac{7}{10}$
p. $\frac{1}{9}$	q. $\frac{7}{11}$	r. $\frac{4}{7}$	s. $\frac{2}{5}$	t. $\frac{2}{3}$

Answers

a. 0.9091, R b. 0.6, T c. 1.2727, R d. 0.4286, R e. 1.4, T f. 1.5, T g. 1.3, T h. 1.3333, R i. 0.1429, R j. 0.7143 , R k. 0.7, T l. 1.6, T m. 0.1818, R n. 0.2857, R o. 1.7, T p. 0.1111, R q. 0.6364, R r. 0.5714, R s. 0.4, T t. 0.6667, R

Helpful Information

A **recurring decimal** is one where the digits after the decimal point repeat indefinitely in some fixed pattern. For example, 0.333… is a recurring decimal, as is 1.2517676767…
A **terminating decimal** has a finite number of decimal places. Its digits do not go on forever. For example, 0.2, 0.33 and 24.163 are terminating decimals.

Recall that a division can be written as a fraction. This means that any fraction can be interpreted as a division.

Strategy for converting a fraction to a decimal
If a simpler conversion isn't obvious…

1. Re-write the fraction (part) as a division
2. Carry out the division using the division algorithm. If needed, add extra zeros until there are no more remainders (and you have a terminating decimal), or until a clear pattern emerges (and you have a recurring decimal)

Note: When calculating the division work to 5 decimal places so that when you give your answer to 4 decimal places your final digit is correct if rounding is required.

Examples

Question: Express $\frac{10}{11}$ as a decimal. Giving your answer to 4 decimal places if appropriate. State whether the decimal is recurring (R) or terminating (T)
Thought process: Using the above strategy we have…

$\frac{10}{11} = 10 \div 11$

$$11 \overline{)\,10.{}^{10}0{}^{1}0{}^{10}0{}^{1}0{}^{10}0} = 00.90909\ldots$$

$\frac{10}{11} \approx 0.9091$

Since the division will go on forever in a pattern 0.9090… is a recurring decimal.
Answer: 0.9091, R

Question: Express $\frac{3}{5}$ as a decimal. Giving your answer to 4 decimal places if appropriate. State whether the decimal is recurring (R) or terminating (T)
Thought process: Using the above strategy we have…

$\frac{3}{5} = \frac{6}{10} = 0.6$ Terminating

Since the decimal has a finite number of decimal places 0.6 a terminating decimal.
Answer: 0.6, T

7 multiplying decimals

Questions Part 1 of 2 – Multiplying large numbers

7.1 Complete the following questions. The expected detail in your working is demonstrated in the example on the right.

a.	256×47	b.	185×23	c.	12×376	d.	269×46
e.	39×268	f.	18×485	g.	49×188	h.	567×15
i.	488×46	j.	22×585	k.	35×556	l.	49×177
m.	23×457	n.	196×22	o.	43×287	p.	17×79
q.	36×255	r.	48×465	s.	576×54	t.	258×46

Answers

a. 12032 b. 4255 c. 4512 d. 12374 e. 10452 f. 8730 g.9212 h. 8505 i. 22448
j. 12870 k. 19460 l. 8673 m. 10511 n. 4312 o. 12341 p. 1343 q. 9180 r. 22320
s. 31104 t. 11868

Helpful Information

Strategy to Multiply Large Numbers

1. Write the biggest number on top of the smallest number, so digits with the same place value are lined up. Draw a multiplication sign on the left and a horizontal line under the sum
2. Start by multiplying the bottom ones digit and the top ones digit
   a. If the answer is less than 10 write the answer directly underneath the ones column
   b. If the answer is greater than 10, write the ones value of the answer directly underneath the ones column and "carry" the tens value on top of the tens column
3. Keep multiplying the bottom ones digit by every other digit in the top number (after ones do tens, then hundreds etc). If a number has been "carried" you will need to add this on **after** you've done the multiplication.
4. For each extra digit in the bottom number you will need to create another answer row. Before carrying out the multiplication you must include a zero(s) in the smaller place value column(s) of the answer row(s)
5. Add up your answers to each multiplication to get your final answer

Example

Question: Calculate 256×47

Thought process: Using the above strategy we have…

```
   2 2
   3 4
   2 5 6
×    4 7
--------
 1 7 9 2
+1 0,2 4 0
--------
 1 2 0 3 2
```

Answer: 12032

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Questions Part 2 of 2 – Multiplying decimals

7.2 Complete the following questions. The expected detail in your working is demonstrated in the example on the right.

a.	1.8×5.77	b.	21×18.7	c.	42×0.499	d.	0.27×0.297
e.	1.3×38.9	f.	0.66×43	g.	0.298×54	h.	1.1×2.68
i.	0.458×5.3	j.	54×28.7	k.	11×0.598	l.	19.6×5.2
m.	0.075×5.5	n.	35×0.257	o.	26.7×27	p.	3.66×51
q.	4.5×39.5	r.	59×0.076	s.	3.68×5.5	t.	0.29×0.357

Answers

a. 10.386 b. 392.7 c. 20.958 d. 0.08019 e. 50.57 f. 28.38 g. 16.092 h. 2.948
i. 2.4274 j. 1549.8 k. 6.578 l. 101.92 m. 0.4125 n. 8.995 o. 720.9 p. 186.66
q. 177.75 r. 4.484 s. 20.24 t. 0.10353

Helpful Information

Strategy to Multiply Decimals

1. Multiply numbers as if the decimal point(s) aren't there.
2. Re-write your whole number answer with as many decimal places as there were in total between the numbers in the original question
3. If possible, check if answer seems reasonable

Example

Question: Calculate 1.8×5.77

Thought process: Using the above strategy we have…

$$\begin{array}{r} {}^{6}5\,{}^{5}7\,7 \\ \times \quad 1\,8 \\ \hline 4\,6\,1\,6 \\ +\ 5{,}7\,7\,0 \\ \hline 1\,0\,3\,8\,6 \end{array}$$

$1.\underline{8} \times 5.\underline{7}\underline{7} = 10.386$

$1.8 \times 5.77 \approx 2 \times 6$
$= 12$

So answer is reasonable

Answer: 10.386

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8 increasing and decreasing by a percentage

8.1 Calculate the following.

a. Increase 87 by 60%	b. Decrease 81 by 7%	c. Increase 24 by 8%	d. Increase 99 by 60%
e. Increase 38 by 90%	f. Decrease 41 by 7%	g. Decrease 32 by 7%	h. Increase 89 by 70%
i. Decrease 32 by 70%	j. Increase 21 by 70%	k. Decrease 25 by 80%	l. Increase 29 by 3%
m. Increase 86 by 6%	n. Increase 59 by 8%	o. Decrease 42 by 2%	p. Decrease 94 by 90%
q. Decrease 69 by 8%	r. Increase 31 by 7%	s. Decrease 77 by 7%	t. Increase 83 by 70%

Answers

a. 139.2 b. 75.33 c. 25.92 d. 158.4 e. 72.2 f. 38.13 g. 29.76 h. 151.3 i. 9.6 j. 35.7 k. 5 l. 29.87 m. 91.16 n. 63.72 o. 41.16 p. 9.4 q. 63.48 r. 33.17 s. 71.61 t. 141.1

Helpful Information

When thinking about a percentage increase or decrease it is helpful to think about the total percentage we will have at the end of the process. For example…

If we increase a number, by say 30%, then we end up with 130% of the original number (100%+30%=130%).

If we decrease a number, by say 30%, then we end up with 70% of the original number (100%-30%=70%)

Strategy for Finding any Percentage of a Number

1. Convert the percentage to a decimal
2. Multiply the number by the decimal

Examples

Question: Increase 87 by 60%

Thought process: Increasing a number by 60% will result in us having 160% of the number (100%+60%=160%). Converting 160% to a decimal gives 1.6. Then using the above strategy we have…

$$160\% \text{ of } 87 = 1.6 \times 87 = 139.2$$

$$\begin{array}{r} {}^{4}87 \\ \times\ 16 \\ \hline 522 \\ +\ 870 \\ \hline 1392 \end{array}$$

Answer: 139.2

Question: Decrease 81 by 7%

Thought process: Decreasing a number by 7% will result in us having 93% of the number (100%-7%=93%). Converting 93% to a decimal gives 0.93. Then using the above strategy we have…

$$93\% \text{ of } 81 = 0.93 \times 81 = 75.33$$

$$\begin{array}{r} {}^{2}93 \\ \times\ 81 \\ \hline 93 \\ +\ 7440 \\ \hline 7533 \end{array}$$

Answer: 75.33

9 simplifying algebraic expressions

Questions Part 1 of 2 – Adding Like Terms

9.1 Simplify the following.

a.	$-5b-3+b$	b.	$-3-4-5x$	c.	$-5-4a+3a$
d.	$2-5a+3a$	e.	$-6-2y-5$	f.	$6c+5c+5$
g.	$-5y+6y+6$	h.	$3y-6y-2y$	i.	$4c-c-2$
j.	$-3z-2+5z$	k.	$-7+2a-5a$	l.	$4y+1+6$
m.	$-2-6b-2$	n.	$-2+5-7x$	o.	$-7-2a-7$
p.	$-3c+3c+5$	q.	$3-3+a$	r.	$-1-6y+2y$
s.	$2b-3+2$	t.	$-2a-5a-7a$	t.	$-2a-5a-7a$

Answers

a. $-4b-3$ b. $-5x-7$ c. $-a-5$ d. $-2a+2$ e. $-2y-11$ f. $11c+5$
g. $y+6$ h. $-5y$ i. $3c-2$ j. $2z-2$ k. $-3a-7$ l. $4y+7$ m. $-6b-4$
n. $-7x+3$ o. $-2a-14$ p. 5 q. a r. $-4y-1$ s. $2b-1$ t. $-14a$

Helpful Information

Like terms involve some combination of pronumerals and numbers where each pronumeral appears with the same power. For example, $3x$, x and $-2x$ are like terms. $4x^3y$, $-x^3y$ and yx^3 are also like terms. Though $3x$ and x^3 are not like terms and neither are $3x$ and $3y$.

When an expression involves addition or subtraction of terms the expression can be **simplified** by combining like terms. For example, $-3x+5$ is in simplified form while $4x+5-7x$ is not in simplified from.

- When simplifying expressions, we need to think flexibly between the ideas of subtraction and negative numbers
- When no coefficient is given the coefficient is 1
  - For example, $b = 1b$ and $-b = -1b$
- It is convention to write the number before the pronumeral.
  - For example, we write $4b$ not $b4$
- It is convention to write simplified expressions with terms involving pronumerals before those without pronumerals
  - For example, we'd write $-3b+2$ instead of $2-3b$

Thinking about Simplifying Expressions Involving Addition and Subtraction

- Identify like terms using coloured circles (circle the operation in front of each term as this is the terms sign)
- Combine like terms by adding or subtracting as indicated by the sign/operation in front of it.

Example

Question: Simplify $-5b-3+b$

Thought process: Using the thought process above we identify $-5b$ and $+b$ as like terms that need to be combined.

$$-5b+b=-5b+1b=-4b.$$

Since it is convention to write terms involving pronumerals first our final answer is $-4b-3$

Answer: $-4b-3$

$-5b - 3 + 1b$

$= -4b - 3$

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Questions Part 2 of 2 – Multiplying Algebraic Terms

9.2 Simplify the following.

a.	$-5z \times -6$	b.	$5a \times -4a$	c.	$5 \times 5c$	d.	$5z \times -5z$
e.	$-3c \times -2c$	f.	$-7y \times 4y$	g.	$-3 \times -3b$	h.	$-4c \times -3c$
i.	$-7c \times -6$	j.	$-2a \times -4a$	k.	$-4c \times -2$	l.	$-z \times z$
m.	$3 \times -2a$	n.	$2 \times -2c$	o.	$-7x \times -7x$	p.	$4 \times 3a$
q.	$-5 \times b$	r.	$2 \times 6a$	s.	$-5c \times -7c$	t.	$4y \times -5$

Answers

a. $30z$ b. $-20a^2$ c. $25c$ d. $-25z^2$ e. $6c^2$ f. $-28y^2$ g. $9b$ h. $12c^2$ i. $42c$ j. $8a^2$ k. $8c$ l. $-z^2$ m. $-6a$ n. $-4c$ o. $49x^2$ p. $12a$ q. $-5b$ r. $12a$ s. $35c^2$ t. $-20y$

Helpful Information

Expressions involving multiplication can also be simplified. We **simplify** expressions involving multiplication by multiplying the coefficients then multiplying the pronumerals. For example, $-12x$ is the simplified form of the expression $4x \times -3x$

Recall…

- When no coefficient is given the coefficient is 1
  - For example, $b = 1b$ and $-b = -1b$
- It is convention to write simplified expressions with terms involving pronumerals before those without pronumerals
  - For example, we'd write $-3b + 2$ instead of $2 - 3b$

Thinking about Simplifying Expressions Involving Multiplication

- If more than one of the terms has a coefficient, multiply the coefficients (the numbers) and write the answer
- If more than one of the terms has a pronumeral, multiply the pronumerals (the letters) and write the answer

Examples

Question: Simplify $-5z \times -6$

Thought process: Using the thought process above we multiply the coefficients -5 and -6 to get 30. Since z is the only pronumeral it does not get multiplied. Giving the final answer of $30z$.

Answer: $30z$

$-5z \times -6$

$30z$

Question: Simplify $5a \times -4a$

Thought process: Using the thought process above we multiply the coefficients 5 and 4 to get -20. We then multiply the pronumerals a and a to get a^2. Giving the final answer of $-20a^2$.

Answer: $-20a^2$

$5a \times -4a$

$-20a^2$

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10 multiplying and dividing algebraic terms

Questions Part 1 of 2 – Applying the index laws to pronumerals

10.1 Simplify the following.

a.	$a^8 \div a^3$	b.	$z^6 \times z$	c.	$(k^3)^2$	d.	$\frac{r^7}{r^6}$
e.	$t^6 \times t^2$	f.	$g^7 \div g^3$	g.	$\frac{b^6}{b}$	h.	$(x^2)^7$
i.	$y^5 \times y^2$	j.	$(a^4)^2$	k.	$\frac{t^5}{t^2}$	l.	$x^5 \div x^5$
m.	$d^3 \div d^2$	n.	$(z^6)^2$	o.	$(x^2)^4$	p.	$\frac{s^6}{s^2}$
q.	$\frac{t^4}{t}$	r.	$(g^3)^3$	s.	$\frac{p^4}{p^3}$	t.	$k^2 \times k^5$

Answers

a. a^5 b. z^7 c. k^6 d. r e. t^8 f. g^4 g. b^5 h. x^{14} i. y^7 j. a^8 k. t^3 l. 1 m. d n. z^{12} o. x^8 p. s^4 q. t^3 r. g^9 s. p t. k^7

Helpful Information

Some index laws are listed below.

Index Laws	Powers of One and Zero
• $a^x \times a^y = a^{x+y}$ • $a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}$ • $(a^x)^y = a^{xy}$	• $a^1 = a$ • $a^0 = 1$

Examples

Question: Simplify $a^8 \div a^3$
Thought process: Using the index law for divisions involving the same base we see that we need to subtract the powers. That is, $a^8 \div a^3 = a^{8-3} = a^5$
Answer: a^5

Question: Simplify $z^6 \times z$
Thought process: Using the index law for multiplications involving the same base we see that we need to add the powers. Note that when no power is specified the power is 1. That is, $z^6 \times z = z^6 \times z^1 = z^{6+1} = z^7$
Answer: z^7

Question: Simplify $(k^3)^2$
Thought process: Using the index law for a power of a power we see that we need to multiply the powers. That is, $(k^3)^2 = k^{3\times2} = k^6$
Answer: k^6

Questions Part 2 of 2 – More complex applications of the index laws to pronumerals

10.2 Simplify the following. Give your answer with positive powers.

a. $\frac{5b^4}{10b^3}$	b. $5x^2 \times 2x^3$	c. $(3x^5)^2$	d. $\frac{2q^4}{q^3}$
e. $4y^3 \times y^3$	f. $4y \times y^5$	g. $\frac{8a^7}{12a^5}$	h. $3y^3 \times 2y^2$
i. $(3r^3)^3$	j. $\frac{16a^8}{20a^7}$	k. $(s^5)^3$	l. $\frac{3b^6}{3b^5}$
m. $2y \times 5y^4$	n. $4y^3 \times 2y^4$	o. $(2s^3)^2$	p. $\frac{20b^7}{20b}$
q. $\frac{20b^4}{12b^3}$	r. $3y \times 3y^2$	s. $y^4 \times 5y^3$	t. $3x^5 \times 4x^5$

Answers

a. $\frac{b}{2}$ b. $10x^5$ c. $9x^{10}$ d. $2q$ e. $4y^6$ f. $4y^6$ g. $\frac{2a^2}{3}$ h. $6y^5$ i. $27r^9$ j. $\frac{4a}{5}$ k. s^{15} l. b m. $10y^5$ n. $8y^7$ o. $4s^6$ p. b^6 q. $\frac{5b}{3}$ r. $9y^3$ s. $5y^7$ t. $12x^{10}$

Helpful Information

Thinking about Simplifying Expressions Involving Multiplication and Division

- If more than one of the terms has a coefficient;
  - Multiply them or
  - Simplify the fraction if possible
- If a pronumeral appears more than once;
  - Multiply the pronumerals (the letters) and write the answer or
  - Divide the numerator and denominator by the term with the smallest power
- If a term is raised to a power
  - Raise the coefficient to the power and then raise the pronumeral to the power

Note: Powers and coefficients of 1 should not be included in final answers.

Examples

Question: Simplify $\frac{5b^4}{10b^3}$. Give your answers with positive powers.
Thought process: Using ideas from above we simplify the fraction $\frac{5}{10}$ to $\frac{1}{2}$ and then divide the numerator and denominator by b^3. This results in an answer of $\frac{b}{2}$.
Answer: $\frac{b}{2}$

Question: Simplify $5x^2 \times 2x^3$. Give your answers with positive powers.
Thought process: Using ideas from above we multiply the coefficients $5 \times 2 = 10$ and then multiply the pronumerals $x^2 \times x^3 = x^5$. Combining these gives $10x^5$
Answer: $10x^5$

Question: Simplify $(3x^5)^2$. Give your answers with positive powers.
Thought process: Using ideas from above we raise 3 to the power of 2, that is $3^2 = 9$ and raise the pronumeral to the power of 2 as well. $(x^5)^2 = x^{10}$. Combining these gives $9x^{10}$
Answer: $9x^{10}$

11 expanding

Questions Part 1 of 2 – Expanding Brackets

11.1 Expand the following expressions.

a.	$4(3x-5)$	b.	$3(2x-6)$	c.	$4(3x+5)$	d.	$3(2x-2)$
e.	$4(x+4)$	f.	$3(2x+5)$	g.	$2(6x-6)$	h.	$3(2x+4)$
i.	$2(4x+6)$	j.	$2(5x-5)$	k.	$3(2x-4)$	l.	$3(3x-5)$
m.	$2(3x-1)$	n.	$3(4x+1)$	o.	$4(x+5)$	p.	$2(x+5)$
q.	$4(4x+1)$	r.	$3(6x-5)$	s.	$2(2x-1)$	t.	$3(4x-2)$

Answers

a. 12x-20 b. 6x-18 c. 12x+20 d. 4x+16 e. 6x+16 f. 6x+15 g. 12x-12 h. 6x+12 i. 8x+12
j. 10x-10 k. 6x-12 l. 9x-15 m. 6x-2 n. 12x+3 o. 4x+20 p. 2x+10
q. 16x+4 r. 18x-15 s. 4x-2 t. 12x-2

Helpful Information

Expanding is the process of writing an expression without brackets. The distributive law is key to this process. The distributive law states that

$$a(b+c)=ab+ac$$

Often in mathematics you will need to think flexibly about the ideas of subtraction and negative numbers.
For example, $a(b-c)=a(b+{}^{-}c)$.
Applying the distributive law we have $a(b+{}^{-}c)=ab+a({}^{-}c)=ab-ac$.
This means that we can use the distributive law for sums and differences.

Distributive Law(s)	
$a(b+c)=ab+ac$	$a(b-c)=ab-ac$

Example

Question: Expand $4(3x-5)$
Thought process: Using the distributive law we have…

$$\begin{aligned}4(3x-5)&=4\times 3x-4\times 5\\&=12\text{x}-20\end{aligned}$$

Answer: $12x-20$

Questions Part 2 of 2 – Expanding Brackets involving Integers

11.2 Expand the following expressions.

a.	$-11(3x-5)$	b.	$-9(3x+3)$	c.	$12(x+5)$	d.	$9(5x-3)$
e.	$-8(2x-2)$	f.	$-7(2x-1)$	g.	$-5(4x+2)$	h.	$10(x+1)$
i.	$-4(2x+4)$	j.	$10(3x+4)$	k.	$-2(4x+4)$	l.	$7(2x+3)$
m.	$-3(4x-2)$	n.	$-4(2x-4)$	o.	$-2(5x+1)$	p.	$8(4x-1)$
q.	$3(5x-3)$	r.	$3(3x-4)$	s.	$9(5x-5)$	t.	$-12(2x-1)$

Answers

a. $-33x+55$ b. $-27x-27$ c. $12x+60$ d. $45x-27$ e. $-16x+16$ f. $-14x+7$
g. $-20x-10$ h. $10x+10$ i. $-8x-16$ j. $30x+40$ k. $-8x-8$ l. $14x+21$
m. $-12x+6$ n. $-8x+16$ o. $-10x-2$ p. $32x-8$ q. $15x-9$ r. $9x-12$
s. $45x-45$ t. $-24x+12$

Example

Question: Expand $-11(3x-5)$
Thought process: Using the distributive law we have…

$$\begin{aligned}-11(3x-5)&=-11\times 3x+{}^{-}11\times{}^{-}5\\&=-33\text{x}+55\end{aligned}$$

Answer: $-33x+55$

12 Expanding and Factorising

Questions Part 1 of 2 – Factorising Algebraic Expressions

12.1 Factorise the following expressions.

a. $16x - 20$	b. $4x + 6$	c. $6x - 12$	d. $3x + 9$
e. $20x - 16$	f. $16x + 16$	g. $6x + 20$	h. $4x + 4$
i. $15x - 15$	j. $10x - 2$	k. $8x + 10$	l. $12x - 12$
m. $12x + 6$	n. $4x + 12$	o. $15x + 18$	p. $9x + 15$
q. $20x - 20$	r. $12x + 24$	s. $16x + 8$	t. $15x - 12$

Answers

a. 4(4x-5) b. 2(2x+3) c. 6(x-2) d. 3(x+3) e. 4(5x-4) f. 16(x+1) g. 2(3x+10)
h. 4(x+1) i. 15(x-1) j. 2(5x-1) k. 2(4x+5) l. 12(x-1) m. 6(2x+1) n. 4(x+3)
o. 3(5x+6) p. 3(3x+5) q. 20(x-1) r. 12(x+2) s. 8(2x+1) t. 3(5x-4)

Helpful Information

Distributive Law
$$a(b + c) = ab + ac$$

The distributive law can be used "in reverse" to write algebraic expressions where the terms have a common factor in factorised form. **Factorised form** is an expression where the common factors of a collection of terms is written at the front of a bracket.

Example

Question: Factorise $16x - 20$

Thought process: The highest common factor of 16 and 20 is 4. This means we can write $16x - 20$ as $4(4x - 5)$. Note that to work out the numbers inside the bracket we divide each term by 4. $16x \div 4 = 4x$ and $20 \div 4 = 5$ and so the answer is $4(4x - 5)$

Answer: $4(4x - 5)$

Questions Part 2 of 2 – Factorising Algebraic Expressions involving Integers

12.2 Factorise the following expressions.

a. $-9 + 18x$	b. $50 - 40x$	c. $-12 - 9x$	d. $60 - 12x$
e. $-6x + 6$	f. $40 - 8x$	g. $-3 - 3x$	h. $-10x - 10$
i. $6x - 8$	j. $-6 - 3x$	k. $-21 + 14x$	l. $32 - 40x$
m. $-3x - 6$	n. $15x - 6$	o. $12 - 30x$	p. $10 - 6x$
q. $-10 - 30x$	r. $10 - 15x$	s. $-24 + 30x$	t. $4 - 4x$

Answers

a. $-9(1 - 2x)$ b. $10(5 - 4x)$ c. $-3(4 + 3x)$ d. $12(5 - x)$ e. $-6(x - 1)$ f. $8(5 - x)$
g. $-3(1 + x)$ h. $-10(x + 1)$ i. $2(3x - 4)$ j. $-3(2 + x)$ k. $-7(3 - 2x)$ l. $8(4 - 5x)$
m. $-3(x + 2)$ n. $3(5x - 2)$ o. $6(2 - 5x)$ p. $2(5 - 3x)$ q. $-10(1 + 3x)$
r. $5(2 - 3x)$ s. $-6(4 - 5x)$ t. $4(1 - x)$

Helpful Information

It is convention for the first term inside a bracket to be positive and so if the first number is negative the number out the front of the bracket will be negative too.

Example

Question: Factorise $-9 + 18x$

Thought process: The highest common factor of 9 and 18 is 9. So we can write $-9 + 18x$ as $-9(1 - 2x)$. Note that to work out the numbers inside the bracket we divide each term by -9.

Answer: $-9(1 - 2x)$

13 substituting into ax+b and ax-b

13.1 Evaluate the following expressions after substituting the given value for x. Note that the expected detail in your working is demonstrated in the example on the right.

a.	$2x+7, x=9$	b.	$2x-1, x=2$	c.	$6x-10, x=3$	d.	$2x+3, x=4$
e.	$4x+5, x=4$	f.	$7x-24, x=6$	g.	$2x+2, x=5$	h.	$2x-4, x=3$
i.	$9x-6, x=3$	j.	$6x-12, x=4$	k.	$4x+8, x=3$	l.	$5x-2, x=2$
m.	$4x-11, x=5$	n.	$4x-1, x=6$	o.	$8x-39, x=6$	p.	$3x-7, x=5$
q.	$3x-17, x=6$	r.	$4x+2, x=6$	s.	$7x-20, x=6$	t.	$9x-23, x=3$

Answers

a. 25 b. 3 c. 8 d. 11 e. 21 f. 18 g. 12 h. 2 i. 21 j. 12 k. 20 l. 8 m. 9 n. 23 o. 9 p. 8 q. 1 r. 26 s. 22 t. 4

Example

Question: Substitute $x=9$ into $2x+7$ and evaluate

Thought process: Following the instructions given in the question we have…

$$2x+7, \quad x=9$$
$$=2\times 9+7$$
$$=18+7$$
$$=25$$

Answer: 25

14 substitution of a whole number into a 2-step expression

14.1 Evaluate the following expressions after substituting the given value for x. The expected detail in your working is demonstrated in the examples on the right.

a. $\frac{x-5}{4}, x=29$	b. $5(x+4), x=7$	c. $2x+7, x=9$	d. $6x-13, x=5$
e. $\frac{x}{9}+6, x=18$	f. $\frac{x-9}{2}, x=13$	g. $\frac{x+21}{6}, x=33$	h. $5(x+2), x=3$
i. $9(x-2), x=4$	j. $5x-3, x=5$	k. $\frac{x+39}{6}, x=15$	l. $4(x+5), x=2$
m. $\frac{x}{2}-2, x=8$	n. $2(x-2), x=3$	o. $\frac{x}{9}-1, x=63$	p. $\frac{x-6}{6}, x=60$
q. $3x+2, x=6$	r. $5x+7, x=5$	s. $\frac{x+22}{7}, x=27$	t. $\frac{x+2}{5}, x=8$

Answers

a. 6 b. 55 c. 25 d. 17 e. 8 f. 2 g. 9 h. 25 i. 18 j. 22 k. 9 l. 28 m. 2 n. 2 o. 6 p. 9 q. 20 r. 32 s. 7 t. 2

Helpful Information

Substitution is the process of replacing a pronumeral with a specified value. It could be helpful to think of a substitution like a swap, just like a substitution in basketball.

Example

Question: Substitute $x = 29$ into $\frac{x-5}{4}$ and evaluate.

Thought process: Following the instructions given in the question we have…

$$\frac{x-5}{4}, \quad x=29$$

$$=\frac{29-5}{4}$$

$$=\frac{24}{4}$$

$$=6$$

Answer: 6

15 substituting an integer into an expression of the form ax+b

15.1 Evaluate the following expressions after substituting the given value for x. The expected detail in your working is demonstrated in the examples on the right.

a.	$-8x-4, x=-9$	b.	$5+4x, x=-1$	c.	$-8x-10, x=-6$	d.	$-6x+1, x=8$
e.	$3-4x, x=-4$	f.	$7-5x, x=4$	g.	$3x-6, x=-5$	h.	$-8x-10, x=1$
i.	$3x-2, x=-3$	j.	$-7x-9, x=-3$	k.	$5-2x, x=7$	l.	$-8x-10, x=-1$
m.	$8-4x, x=-2$	n.	$-9x+4, x=3$	o.	$8x-3, x=-5$	p.	$6x-10, x=-3$
q.	$-6x+6, x=7$	r.	$1-7x, x=7$	s.	$-6x-4, x=-8$	t.	$-8x-10, x=-3$

Answers

a. 68 b. 1 c. 38 d. -47 e. 19 f. -13 g. -21 h. -18 i. -11 j. 12 k. -9 l. -2 m. 16 n. -23 o. -43 p. -28 q. -36 r. -48 s. 44 t. 14

Helpful Information

When substituting anything other than a whole number it is helpful to use brackets when multiplication is involved. This is done to avoid any potential confusion between multiplying by a negative number and subtracting.

Example

Question: Substitute $x=-9$ into $-8x-4$ and evaluate.

Thought process: Following the instructions given in the question we have…

$$-8x-4, \quad x=-9$$
$$=-8(-9)-4$$
$$=72-4$$
$$=68$$

Answer: 68

16 Substituting a fraction into an expression of the form $ax+b$

16.1 Evaluate the following expressions after substituting the given value for x. The expected detail in your working is demonstrated in the examples on the right.

a. $-2x-2, x=\frac{3}{7}$ b. $-4x-4, x=\frac{1}{9}$ c. $2x-4, x=\frac{1}{2}$ d. $5x-5, x=\frac{2}{3}$

e. $-5x-2, x=\frac{2}{3}$ f. $-5x-2, x=\frac{1}{3}$ g. $5x-1, x=\frac{4}{5}$ h. $4-3x, x=\frac{7}{12}$

i. $4-2x, x=\frac{2}{9}$ j. $3x-4, x=\frac{1}{4}$ k. $-5x+1, x=\frac{2}{9}$ l. $-3x-4, x=\frac{4}{5}$

m. $-5x-4, x=\frac{2}{3}$ n. $4x-5, x=\frac{1}{8}$ o. $3x-2, x=\frac{1}{3}$ p. $4+2x, x=\frac{2}{5}$

q. $3+4x, x=\frac{2}{3}$ r. $4+2x, x=\frac{2}{3}$ s. $1+2x, x=\frac{4}{5}$ t. $5x+3, x=\frac{2}{5}$

Answers

a. $-\frac{20}{7}$ or $-2\frac{6}{7}$ b. $-\frac{40}{9}$ or $-4\frac{4}{9}$ c. -3 d. $-\frac{5}{3}$ or $-1\frac{2}{3}$ e. $-\frac{16}{3}$ or $-5\frac{1}{3}$ f. $-\frac{11}{3}$ or $-3\frac{2}{3}$ g. 3 h. $\frac{9}{4}$ or $2\frac{1}{4}$ i. $\frac{32}{9}$ or $3\frac{5}{9}$ j. $-\frac{13}{4}$ or $-3\frac{1}{4}$ k. $-\frac{1}{9}$ l. $-\frac{32}{5}$ or $-6\frac{2}{5}$ m. $-\frac{22}{3}$ or $-7\frac{1}{3}$ n. $-\frac{9}{2}$ or $-4\frac{1}{2}$ o. -1 p. $\frac{24}{5}$ or $4\frac{4}{5}$ q. $\frac{17}{3}$ or $5\frac{2}{3}$ r. $\frac{16}{3}$ or $5\frac{1}{3}$ s. $\frac{13}{5}$ or $2\frac{3}{5}$ t. 5

Helpful Information

When substituting anything other than a whole number it is helpful to use brackets when multiplication is involved. This is done to avoid any potential confusion between multiplying by a negative number and subtracting.

Example

Question: Substitute $x=\frac{3}{7}$ into $-2x-2$ and evaluate.

Thought process: Following the instructions given in the question we have…

$$-2x-2, \quad x=\frac{3}{7}$$
$$=-2\left(\frac{3}{7}\right)-2$$
$$=-\frac{2}{1}\times\frac{3}{7}-2$$
$$=-\frac{6}{7}-2$$
$$=-\frac{6}{7}-\frac{2}{1}$$
$$=-\frac{6}{7}-\frac{14}{7}$$
$$=\frac{-20}{7} \quad \left(\text{or } -2\frac{6}{7}\right)$$

Answer: $-\frac{20}{7}$ (or $-2\frac{6}{7}$)

17 solving 2-step equations of the form ax+b=c and ax-b=c

17.1 Solve the following equations and include appropriate line by line working out. If an answer says $x = 6$ and your answer is in the form $6 = x$ don't worry as both are correct. Note that the expected detail in your working is demonstrated in the examples on the right.

a.	$37 = 10 + 9x$	b.	$5x - 2 = 28$	c.	$3 = 4x - 13$	d.	$15 = 9 + 3x$
e.	$8x + 2 = 58$	f.	$37 = 6x - 5$	g.	$11 = 9x - 43$	h.	$6 = 3x - 9$
i.	$41 = 5x + 6$	j.	$3x + 2 = 14$	k.	$2x - 2 = 8$	l.	$3x + 9 = 30$
m.	$9 = 5x - 1$	n.	$6x + 5 = 23$	o.	$8x - 12 = 4$	p.	$5 = 3x - 1$
q.	$35 = 8x - 5$	r.	$17 = 9 + 4x$	s.	$8x - 39 = 9$	t.	$2x + 3 = 7$

Answers

a. 27=9x, x=3 b. 5x=30, x=6 c. 16=4x, x=4 d. 6=3x, x=2 e. 8x=56, x=7 f. 42=6x, x=7 g. 54=9x, x=6 h. 15=3x, x=5 i. 35=5x, x=7 j. 3x=12, x=4 k. 2x=10, x=5 l. 3x=21, x=7 m. 10=5x, x=2 n. 6x=18, x=3 o. 8x=16, x=2 p. 6=3x, x=2 q. 40=8x, x=5 r. 8=4x, x=2 s. 8x=48, x=6 t. 2x=4, x=2

Helpful Information

Solving an algebraic equation is the process of finding the value of the unknown which makes the equation true.

Thinking about Solving

If the unknown appears once you could solve the equation by:

1. Using opposite operations ($+/-$ and $\times/\div$) to undo the furthest removed operation
2. Doing the same action to both sides to keep the equation true

"Appropriate line by line working" requires you to

- Have the unknown appear in each line of working
- Move **exactly one** step closer to isolating the unknown with each line

Examples

Question: Solve $37 = 10 + 9x$ and include appropriate line by line working out.

Thought process: Using the above thought process we identify multiplying by 9 as the closest operation and adding 10 as the furthest removed. Subtracting 10 from both sides gives $27 = 9x$. Dividing both sides by 3 gives $3 = x$

Answer: $x = 3$

	$37 = 10 + 9x$
-10	$27 = 9x$
$\div 3$	$3 = x$

Question: Solve $5x - 2 = 28$ and include appropriate line by line working out.

Thought process: Using the above thought process we identify multiplying by 5 as the closest operation and subtracting 2 as the furthest removed. Adding 2 to both sides gives $5x = 30$. Dividing both sides by 5 gives $x = 6$

Answer: $x = 6$

	$5x - 2 = 28$
$+2$	$5x = 30$
$\div 5$	$x = 6$

18 solving 2-step equations with whole number solutions

18.1 Solve the following equations and include appropriate line by line working out. If an answer says $x = 6$ and your answer is in the form $6 = x$ don't worry as both are correct. The expected detail in your working is demonstrated in the examples on the right.

a. $8 = \frac{x}{5} - 2$	b. $2(x+4) = 28$	c. $7 = \frac{x-5}{3}$	d. $2x + 4 = 28$
e. $11 = 7x - 17$	f. $9 = \frac{x+15}{3}$	g. $13 = 7 + 3x$	h. $40 = 9x + 4$
i. $0 = \frac{x}{3} - 5$	j. $6 = \frac{x}{9} + 3$	k. $4 = 9x - 59$	l. $37 = 4x + 9$
m. $15 = 2x + 9$	n. $30 = 3(x+4)$	o. $9 = \frac{x-6}{4}$	p. $\frac{x-4}{2} = 7$
q. $4x - 12 = 0$	r. $7(x+5) = 84$	s. $8 + 9x = 62$	t. $3 = \frac{x}{7} + 2$

Answers

a. $10 = \frac{x}{5},\ x = 50$ b. $x + 4 = 14,\ x = 10$ c. $21 = x - 5,\ x = 26$
d. $2x = 24,\ x = 12$ e. $28 = 7x,\ x = 4$ f. $27 = x + 15,\ x = 12$ g. $3x = 6,\ x = 2$
h. $36 = 9x,\ x = 4$ i. $5 = \frac{x}{3}, x = 15$ j. $3 = \frac{x}{9},\ x = 27$ k. $63 = 9x,\ x = 7$
l. $28 = 4x,\ x = 7$ m. $6 = 2x,\ x = 3$ n. $10 = x + 4,\ x = 6$ o. $36 = x - 6,\ x = 42$
p. $x - 4 = 14,\ x = 18$ q. $4x = 12,\ x = 3$ r. $x + 5 = 12,\ x = 7$ s. $9x = 54,\ x = 6$
t. $1 = \frac{x}{7},\ x = 7$

Helpful Information

> Solving an algebraic equation is the process of finding the value of the unknown which makes the equation true.

Thinking about Solving

If the unknown appears once you could solve the equation by:

1. Using opposite operations ($+/-$ and $\times/\div$) to undo the furthest removed operation
2. Doing the same action to both sides to keep the equation true

"Appropriate line by line working" requires you to

- Have the unknown appear in each line of working
- Move **exactly one** step closer to isolating the unknown with each line

Examples

Question: Solve $8 = \frac{x}{5} - 2$ and include appropriate line by line working out.

Thought process: Using the above thought process we identify dividing by 5 as the closest operation and subtracting 2 as the furthest removed. Adding 2 to both sides gives $10 = \frac{x}{5}$. Multiplying both sides by 5 gives $50 = x$

$8 = \frac{x}{5} - 2$
$+2 \quad 10 = \frac{x}{5}$
$\times 5 \quad 50 = x$

Answer: $10 = \frac{x}{5},\ x = 50$

Question: Solve $2(x+4) = 28$ and include appropriate line by line working out.

Thought process: Using the above thought process we identify adding 4 as the closest operation and multiplying by 2 as the furthest removed. Dividing both sides by 2 gives $x + 4 = 14$. Subtracting 4 from both sides gives $x = 10$

$2(x+4) = 28$
$\div 2 \quad x + 4 = 14$
$-4 \quad x = 10$

Answer: $x + 4 = 14,\ x = 10$

19 solving equations where the unknown appears on both sides

19.1 Solve the following equations and include appropriate line by line working out.
Note: If an answer says $x = 6$ and your answer is in the form $6 = x$ don't worry as both are correct.

a. $6x - 15 = 2x - 3$	b. $5x + 15 = 6x + 9$	c. $4x + 5 = 2x + 9$
d. $4x - 5 = 2x - 1$	e. $4x + 3 = 3x + 6$	f. $x + 6 = 6x - 4$
g. $6x + 7 = 2x + 27$	h. $6x - 5 = 5x$	i. $6x + 6 = 4x + 12$
j. $4x + 10 = 3x + 12$	k. $2x - 8 = 5x - 26$	l. $4x + 4 = x + 22$
m. $x - 4 = 4x - 19$	n. $4x + 2 = 3x + 7$	o. $6x - 5 = 5x - 2$
p. $6x - 2 = 2x + 14$	q. $2x + 9 = 4x + 1$	r. $6x + 2 = 2x + 10$
s. $6x - 6 = 4x + 2$	t. $3x - 1 = 2x + 4$	

Answers
a. $4x = 12, x = 3$ b. $15 = x + 9, x = 6$ c. $2x = 4, x = 2$ d. $2x = 4, x = 2$
e. $x + 3 = 6, x = 3$ f. $10 = 5x, x = 2$ g. $4x = 20, x = 5$ h. $x - 5 = 0, x = 5$
i. $2x = 6, x = 3$ j. $x + 10 = 12, x = 2$ k. $18 = 3x, x = 6$ l. $3x = 18, x = 6$
m. $15 = 3x, x = 5$ n. $x + 2 = 7, x = 5$ o. $x - 5 = -2, x = 3$ p. $4x = 16, x = 4$
q. $8 = 2x, x = 4$ r. $4x = 8, x = 2$ s. $2x = 8, x = 4$ t. $x - 1 = 4, x = 5$

Helpful Information
If the unknown appears on both sides of the equation a good first step is to remove the term with the lowest coefficient using opposite operations and doing the same to both sides.

It is also important to note that the operation in front of a constant can be instead read as a sign. For example $\mathrm{x} - 3 = \mathrm{x} + {}^{-}3$

Examples
Question: Solve $6x - 15 = 2x - 3$ and include appropriate line by line working out.
Thought process: Using the above thought process we identify $2x$ as the term with the lowest coefficient. Subtracting $2x$ from both sides gives $4x - 15 = -3$ (where subtract 3 has become negative 3). We then continue using standard methods.

$$\begin{array}{r|l} & 6x - 15 = 2x - 3 \\ -2x & 4x - 15 = -3 \\ +15 & 4x = 12 \\ \div 4 & x = 3 \end{array}$$

Answer: $4x = 12, x = 3$

Question: Solve $5x + 15 = 6x + 9$ and include appropriate line by line working out.
Thought process: Using the above thought process we identify $5x$ as the term with the lowest coefficient. Subtracting $5x$ from both sides gives $15 = x + 9$. We then continue using standard methods.

$$\begin{array}{r|l} & 5x + 15 = 6x + 9 \\ -5x & 15 = x + 9 \\ -9 & 6 = x \end{array}$$

Answer: $15 = 9 + x,\ x = 6$

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20 solving more complex equations

Questions Part 1 of 5 – Solving equations with fractional solutions

20.1 Solve the following equations.

a. $6x = 8$	b. $3x = 2$	c. $2x = 3$	d. $10x = 15$
e. $8x = 7$	f. $9x = 6$	g. $8x = 1$	h. $7x = 9$
i. $2x = 3$	j. $4x = 14$	k. $9x = 23$	l. $8x = 22$
m. $2x = 5$	n. $7x = 20$	o. $3x = 4$	p. $9x = 28$

Answers

a. $x = \frac{4}{3}$ b. $x = \frac{2}{3}$ c. $x = \frac{3}{2}$ d. $x = \frac{3}{2}$ e. $x = \frac{7}{8}$ f. $x = \frac{2}{3}$ g. $x = \frac{1}{8}$ h. $x = \frac{9}{7}$ i. $x = \frac{3}{2}$
j. $x = \frac{7}{2}$ k. $x = \frac{23}{9}$

Helpful Information

When solving one-step equations involving a multiplication, we solve the equation by dividing both sides by the coefficient of the unknown. This is shown in the example below.

$$\begin{array}{r|l} & 8x = 16 \\ \div 8 & x = 2 \end{array}$$

So far, we have only encountered questions where the coefficient divides the other value exactly. If this is not the case, we still solve the equation by dividing both sides by the coefficient, though instead of evaluating the division we leave it as a fraction (and simplify if needed).

Examples

Question: Solve $6x = 8$

Thought process: Using the approach discussed in the above subskill we have..

$$\begin{array}{r|l} & 6x = 8 \\ \div 6 & x = \frac{8}{6} \\ & x = \frac{4}{3} \end{array}$$

Answer: $x = \frac{4}{3}$

Questions Part 2 of 5 – Solving equations involving negative numbers

20.2 Solve the following equations and include appropriate line by line working out. If an answer says $x = 6$ and your answer is in the form $6 = x$ don't worry as both are correct. The expected detail in your working is demonstrated in the examples on the right.

a.	$3x - 6 = 2$	b.	$-17 = -x - 8$	c.	$25 = 4x - 1$
d.	$24 = 9x - 5$	e.	$47 = -8x - 9$	f.	$3x + 1 = 20$
g.	$5x + 9 = -27$	h.	$-28 = 8x - 4$	i.	$4x + 1 = -16$
j.	$46 = 8x + 9$	k.	$5x + 1 = -13$	l.	$-32 = 7x - 4$
m.	$6 = 3x - 5$	n.	$-14 = 5x + 6$	o.	$-6x + 8 = -16$
p.	$-x + 6 = 1$	q.	$-17 = 10x + 2$	r.	$9x + 3 = 34$
s.	$-55 = 8x + 9$	t.	$9x - 1 = 38$	u.	$3 = 4x + 5$

Answers

a. $3x = 8, x = \frac{8}{3}$ or $2\frac{2}{3}$ b. $-9 = -x, x = 9$ c. $26 = 4x, x = \frac{13}{2}$ or $6\frac{1}{2}$
d. $29 = 9x, x = \frac{29}{9}$ or $3\frac{2}{9}$ e. $56 = -8x, x = -7$ f. $3x = 19, x = \frac{19}{3}$ or $6\frac{1}{3}$
g. $5x = -36, x = -\frac{36}{5}$ or $-7\frac{1}{5}$ h. $-24 = 8x, x = -3$
i. $4x = -17, x = -\frac{17}{4}$ or $-4\frac{1}{4}$ j. $37 = 8x, x = \frac{37}{8}$ or $4\frac{5}{8}$
k. $5x = -14, x = -\frac{14}{5}$ or $-2\frac{4}{5}$ l. $-28 = 7x, x = -4$ m. $11 = 3x, x = \frac{11}{3}$ or $3\frac{2}{3}$
n. $-20 = 5x, x = -4$ o. $-24 = -6x, x = 4$ p. $-x = -5, x = 5$
q. $-19 = 10x, x = -\frac{19}{10}$ or $-1\frac{9}{10}$ r. $9x = 31, x = \frac{31}{9}$ or $3\frac{4}{9}$
s. $-64 = 8x, x = -8$ t. $9x = 39, x = \frac{13}{3}$ or $4\frac{1}{3}$ u. $-2 = 4x, \ x = -\frac{1}{2}$

Helpful Information

Solving an algebraic equation is the process of finding the value of the unknown which makes the equation true.

Thinking about Solving

If the unknown appears once you could solve the equation by:

1. Using opposite operations ($+/-$ and $\times/\div$) to undo the furthest removed operation
2. Doing the same action to both sides to keep the equation true

"Appropriate line by line working" requires you to

- Have the unknown appear in each line of working
- Move **exactly one** step closer to isolating the unknown with each line

Examples

Question: Solve $3x - 6 = 2$ and include appropriate line by line working out.

Thought process: Using the above thought process we identify multiplying by 3 as the closest operation and subtracting 6 as the furthest removed. Adding 6 to both sides gives $3x = 8$. Dividing both sides by 3 gives $x = \frac{8}{3}$

$$\begin{array}{r|l} & 3x - 6 = 2 \\ +6 & 3x = 8 \\ \div 3 & x = \frac{8}{3} \end{array}$$

Answer: $3x = 8, x = \frac{8}{3}$

Question: Solve $-17 = -x - 8$ and include appropriate line by line working out.

Thought process: Using the above thought process we identify multiplying by -1 as the closest operation and subtracting 8 as the furthest removed. Adding 8 to both sides gives $-9 = -x$. Since $-x = -1x$ our final step is dividing both sides by -1 to get $9 = x$

$$\begin{array}{r|l} & -17 = -x - 8 \\ +8 & -9 = -x \\ \div -1 & 9 = x \end{array}$$

Answer: $-9 = -x, \ x = 9$

Questions Part 3 of 5 – Solving equations where the unknown is on both sides involving negative coefficients or solutions

20.2 Solve the following equations and include appropriate line by line working out. If an answer says $x = 6$ and your answer is in the form $6 = x$ don't worry as both are correct. The expected detail in your working is demonstrated in the examples on the right.

a. $-5x + 10 = 2x - 4$ b. $-3x - 2 = -7x - 2$ c. $-5x - 4 = -3x + 4$

d. $-x - 8 = 4x + 2$ e. $-5x + 9 = -3x - 7$ f. $-3x + 4 = 2x - 6$

g. $-4x + 1 = -5x + 3$ h. $-2x + 6 = -x + 1$ i. $-6x - 2 = -5x - 1$

j. $-3x - 6 = -2x + 7$ k. $x - 1 = 3x + 5$ l. $-5x + 9 = 3x - 7$

Answers

a. $14 = 7x, x = 2$ b. $0 = 10x, x = 0$ c. $-8 = 2x, x = -4$ d. $-10 = 5x, x = -2$
e. $16 = 2x, x = 8$ f. $10 = 5x, x = 2$ g. $x + 1 = 3, x = 2$ h. $6 = x + 1, x = 5$
i. $-2 = x - 1, x = -1$ j. $-6 = x + 7, x = -13$ k. $-6 = 2x, x = -3$
l. $16 = 8x, x = 2$

Helpful Information

If the unknown appears on both sides of the equation a good first step is to remove the term with the lowest coefficient using opposite operations and doing the same to both sides.

Example

Question: Solve $-5x + 10 = 2x - 4$ and include appropriate line by line working out.

Thought process: Using the above thought process we identify $-5x$ as the term with the lowest coefficient. Adding $5x$ to both sides gives $10 = 7x - 4$. We then continue using standard methods.

Answer: $14 = 7x, x = 2$

$$\begin{array}{r|l} & -5x + 10 = 2x - 4 \\ +5x & 10 = 7x - 4 \\ +4 & 14 = 7x \\ \div 7 & 2 = x \end{array}$$

Question: Solve $-3x - 2 = -7x - 2$ and include appropriate line by line working out.

Thought process: Using the above thought process we identify $-7x$ as the term with the lowest coefficient. Adding $7x$ to both sides gives $4x - 2 = -2$. We then continue using standard methods.

Answer: $4x = 0, x = 0$

$$\begin{array}{r|l} & -3x - 2 = -7x - 2 \\ +7x & 4x - 2 = -2 \\ +2 & 4x = 0 \\ \div 4 & x = 0 \end{array}$$

Questions Part 4 of 5 – Solving equations where the unknown is on both sides and the solution is a fraction

20.4 Solve the following equations and include appropriate line by line working out. If an answer says $x = 6$ and your answer is in the form $6 = x$ don't worry as both are correct. The expected detail in your working is demonstrated in the examples on the right.

a. $-5x + 8 = 7x - 7$	b. $-4x = 5x + 10$	c. $-2x + 1 = 2x + 7$
d. $5x - 5 = x + 20$	e. $-5x + 9 = 2x + 7$	f. $x - 4 = -5x - 2$
g. $-4x + 3 = -2x - 8$	h. $4x - 5 = -x + 7$	i. $2x - 7 = 6x + 4$
j. $-6x - 3 = 6x - 2$	k. $-4x = 3x + 4$	l. $-5x - 9 = 5x - 4$
m. $5x + 5 = -5x - 3$	n. $-3x - 3 = -4x + 2$	o. $3x - 10 = 6x - 3$

Answers

a. $15 = 12x, x = \frac{5}{4}$ b. $-10 = 9x, x = -\frac{10}{9}$ c. $4x = 6,\ x = -\frac{3}{2}$ d. $4x = 25, x = \frac{25}{4}$
e. $2 = 7x, x = \frac{2}{7}$ f. $6x = 2, x = \frac{1}{3}$ g. $11 = 2x, x = \frac{11}{2}$ h. $5x = 12, x = \frac{12}{5}$
i. $-11 = 4x, x = -\frac{11}{4}$ j. $-1 = 12x, x = -\frac{1}{12}$ k. $-4 = 7x, x = -\frac{4}{7}$
l. $-5 = 10x, x = -\frac{1}{2}$ m. $10x = -8, x = -\frac{4}{5}$ n. $x - 3 = 2, x = 5$
o. $-7 = 3x, x = -\frac{7}{3}$

Helpful Information

Remember, if in the final step a division is required and the number does not divide the other exactly, leave the answer as a fraction and simplify if needed.

Example

Question: Solve $-5x + 8 = 7x - 7$and include appropriate line by line working out.

Thought process: We begin as usual, by removing the unknown with the lowest coefficient and then removing the constant term. We then have

$$15 = 12x$$

This can be solved by dividing both sides by 12. Since 12 doesn't divide 15 exactly we leave it as a fraction $15 \div 12 = \frac{15}{12}$ and finalise our answer by simplifying the fraction $\frac{15}{12} = \frac{5}{4}$

$$\begin{array}{r|l} & -5x + 8 = 7x - 7 \\ +5x & 8 = 12x - 7 \\ +7 & 15 = 12x \\ \div 12 & \frac{15}{12} = x \\ & x = \frac{5}{4} \end{array}$$

Answer: $3x = 26, x = \frac{26}{3}$

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Questions Part 5 of 5 – Solving equations where expanding brackets is recommended

20.5 Solve the following equations and include appropriate line by line working out.

If an answer says $x = 6$ and your answer is in the form $6 = x$ don't worry as both are correct. The expected detail in your working is demonstrated in the examples on the right.

a.	$14 = 3(x - 4)$	b.	$6(x + 4) = 15$	c.	$27 = 6(x - 9)$
d.	$5(x - 9) = 21$	e.	$2(x + 8) = 5$	f.	$14 = 3(x - 5)$
g.	$16 = 6(x + 6)$	h.	$5(x - 1) = 23$	i.	$7 = 2(x + 10)$
j.	$3(x - 3) = 10$	k.	$5(x - 9) = 11$	l.	$3(x - 5) = 8$
m.	$14 = 5(x - 1)$	n.	$2(x + 5) = 5$	o.	$26 = 5(x + 5)$
p.	$13 = 5(x - 10)$	q.	$17 = 3(x - 8)$	r.	$17 = 3(x - 9)$
s.	$14 = 3(x - 5)$	t.	$7 = 3(x + 3)$	u.	$5 = 2(x + 3)$

Answers

a. $14 = 3x - 12, x = \frac{26}{3}$ or $8\frac{2}{3}$ b. $6x + 24 = 15, x = -\frac{3}{2}$ or $-1\frac{1}{2}$
c. $27 = 6x - 54, x = \frac{27}{2}$ or $13\frac{1}{2}$ d. $5x - 45 = 21, x = \frac{66}{5}$ or $13\frac{1}{5}$
e. $2x + 16 = 5, x = -\frac{11}{2}$ or $-5\frac{1}{2}$ f. $14 = 3x - 15, x = \frac{29}{3}$ or $9\frac{2}{3}$
g. $16 = 6x + 36, x = -\frac{10}{3}$ or $-3\frac{1}{3}$ h. $5x - 5 = 23, x = \frac{28}{5}$ or $5\frac{3}{5}$
i. $7 = 2x + 20, x = -\frac{13}{2}$ or $-6\frac{1}{2}$ j. $3x - 9 = 10, x = \frac{19}{3}$ or $6\frac{1}{3}$ k. $5x - 45 = 11, x = \frac{56}{5}$ or $11\frac{1}{5}$ l. $3x - 15 = 8, x = \frac{23}{3}$ or $7\frac{2}{3}$ m. $14 = 5x - 5, x = \frac{19}{5}$ or $3\frac{4}{5}$
n. $2x + 10 = 5, x = -\frac{5}{2}$ or $-2\frac{1}{2}$ o. $26 = 5x + 25, x = \frac{1}{5}$
p. $13 = 5x - 50, x = \frac{63}{5}$ or $12\frac{3}{5}$ q. $17 = 3x - 24, x = \frac{41}{3}$ or $13\frac{2}{3}$
r. $17 = 3x - 27, x = \frac{44}{3}$ or $14\frac{2}{3}$ s. $14 = 3x - 15, x = \frac{29}{3}$ or $9\frac{2}{3}$
t. $7 = 3x + 9, x = -\frac{2}{3}$ u. $5 = 2x + 6, x = -\frac{1}{2}$

Helpful Information

Up until now, when we've seen equations involving brackets our first step has been to divide (as the multiplication has been the furthest removed operation). See the example on the right.

$$2(x + 4) = 28$$
$\div 2$ $$x + 4 = 14$$
-4 $$x = 10$$

The equations in this skill all involve a bracket where the number that is in front of the bracket does not divide the number on the other side of the equals sign exactly. For example, $14 = 3(x - 4)$. While it is still possible for the first step to be dividing both sides 3, this introduces a fraction in the first step. Alternatively, if we began by expanding the bracket the equation can be solved without a fraction appearing until the final step (making the solving process a lot easier!).

Thinking about Solving Equations of the form $\boldsymbol{a(x + b) = c}$

- If a divides c exactly, begin by dividing both sides by a.
- If a does not divide c exactly, use the distributive law to expand the brackets and then solve.

Example

Question: Solve $14 = 3(x - 4)$ and include appropriate line by line working out.

Thought process: Since 3 doesn't divide 14 exactly we begin by expanding the bracket. Subtracting 12 is furthest removed so we add 12 to both sides to get $26 = 3x$. We then divide both sides by 3. Dividing both sides by 3 gives $x = \frac{26}{3}$

Answer: $3x = 26, x = \frac{26}{3}$

$$14 = 3(x - 4)$$
expand $$14 = 3x - 12$$
$+\ 12$ $$26 = 3x$$
$\div 3$ $$\frac{26}{3} = x$$
$$(\text{or } 8\tfrac{2}{3})$$

21 completing coordinates

21.1 Complete the missing value in each coordinate so it satisfies the given relationship.

a. $y = 5x + 2,\ (0, \_) \ \& \ (\_, 0)$ b. $y = -5x - 2,\ (0, \_) \ \& \ (\_, 0)$

c. $y = 4x + 1,\ (0, \_) \ \& \ (\_, 0)$ d. $y = 2x - 3,\ (0, \_) \ \& \ (\_, 0)$

e. $y = 3x + 2,\ (0, \_) \ \& \ (\_, 0)$ f. $y = -3x + 2,\ (0, \_) \ \& \ (\_, 0)$

g. $y = -2x + 2,\ (0, \_) \ \& \ (\_, 0)$ h. $y = -3x - 3,\ (0, \_) \ \& \ (\_, 0)$

i. $y = -4x - 3,\ (0, \_) \ \& \ (\_, 0)$ j. $y = 4x - 2,\ (0, \_) \ \& \ (\_, 0)$

k. $y = -x + 3,\ (0, \_) \ \& \ (\_, 0)$ l. $y = 5x + 1,\ (0, \_) \ \& \ (\_, 0)$

m. $y = -3x + 3,\ (0, \_) \ \& \ (\_, 0)$ n. $y = 2x + 1,\ (0, \_) \ \& \ (\_, 0)$

o. $y = 4x + 3,\ (0, \_) \ \& \ (\_, 0)$ p. $y = -2x + 3,\ (0, \_) \ \& \ (\_, 0)$

q. $y = 2x + 2,\ (0, \_) \ \& \ (\_, 0)$ r. $y = -x - 3,\ (0, \_) \ \& \ (\_, 0)$

Answers

a. 2 & $-\frac{2}{5}$ b. -2 & $-\frac{2}{5}$ c. 1 & $-\frac{1}{4}$ d. -3 & $\frac{3}{2}$ e. 2 & $-\frac{2}{3}$ f. 2 & $\frac{2}{3}$ g. 2 & 1 h. -3 & -1
i. -3 & $-\frac{3}{4}$ j. -2 & $\frac{1}{2}$ k. 3 & 3 l. 1 & $-\frac{1}{5}$ m. 3 & 1 n. 1 & $-\frac{1}{2}$ o. 3 & $-\frac{3}{4}$ p. 3 & $\frac{3}{2}$
q. 2 & -1 r. -3 & -3

Helpful Information

Thinking about Coordinates Satisfying a Rule

For a coordinate to satisfy a linear relationship the x and y value must be related just as the rule states.

Strategy for Completing a Coordinate Using a Rule

1. Substitute the value into the rule
2. If
    - The x value has been given → find the y-value by evaluating
    - The y-value has been given → find the y-value by solving the equation

Example

Question: Consider the linear relationship $y = 5x + 2$. Complete the missing value in each coordinate

$$(0, \_) \ \& \ (\_, 0)$$

so each satisfies the rule.

Thought process: Using the above strategy we have

$y = 5x + 2 \qquad (\overset{x}{0}, \_)$

$y = 5 \times 0 + 2$

$y = 0 + 2$

$y = 2$

$y = 5x + 2 \qquad (\_, \overset{y}{0})$

$0 = 5x + 2$

$-2 \mid -2 = 5x$

$\div 5 \mid -\frac{2}{5} = x$

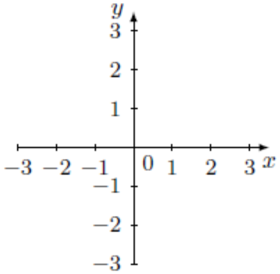
Answer: $(0,2)$ and $(-\frac{2}{5}, 0)$

22 Sketching linear graphs using axis intercepts

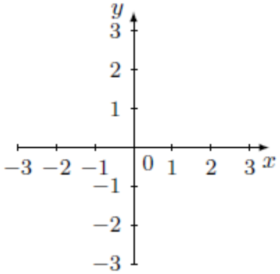
🖶 To be printed or completed using a stylus

22.1 A linear rule and the coordinates of its axis intercepts are given. Use this information to sketch the graph of each linear rule on the Cartesian planes below.

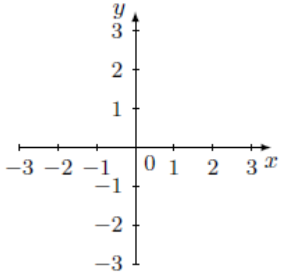
a. $y = 5x + 2$, $(0,2)$ & $\left(-\frac{2}{5}, 0\right)$



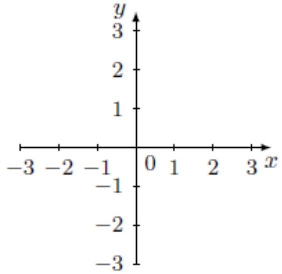
b. $y = -5x - 2$, $(0,-2)$ & $\left(-\frac{2}{5}, 0\right)$



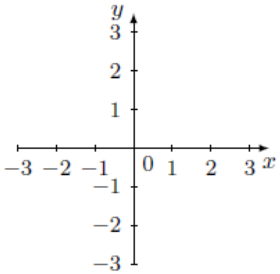
c. $y = 4x + 1$, $(0,1)$ & $\left(-\frac{1}{4}, 0\right)$



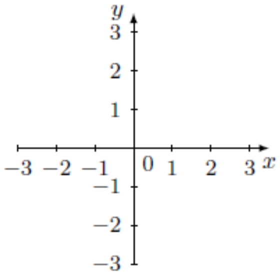
d. $y = 2x - 3$, $(0,-3)$ & $\left(-\frac{3}{2}, 0\right)$



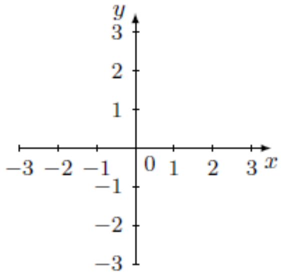
e. $y = 3x + 2$, $(0,2)$ & $\left(-\frac{2}{3}, 0\right)$



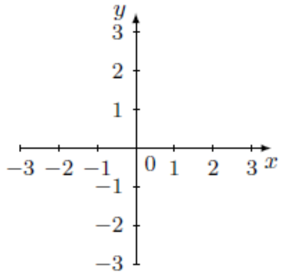
f. $y = -3x + 2$, $(0,2)$ & $\left(\frac{2}{3}, 0\right)$



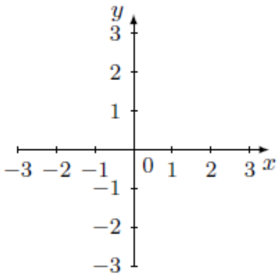
g. $y = -2x + 2$, $(0,2)$ & $(1,0)$



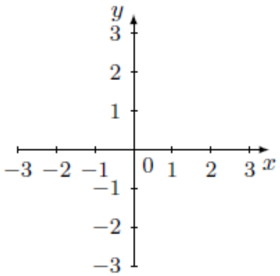
h. $y = -3x - 3$, $(0,-3)$ & $(-1,0)$



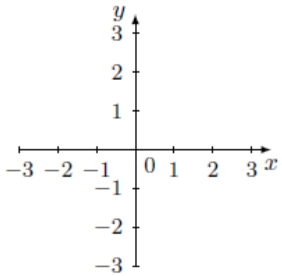
i. $y = -4x - 3$, $(0,-3)$ & $\left(-\frac{3}{4}, 0\right)$



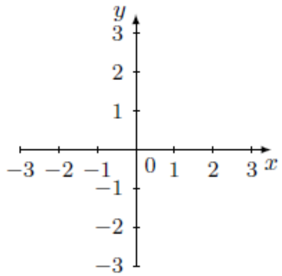
j. $y = 4x - 2$, $(0,-2)$ & $\left(\frac{1}{2}, 0\right)$



k. $y = -x + 3$, $(0,3)$ & $(3,0)$



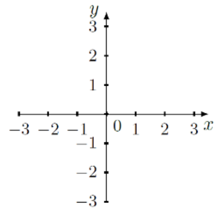
l. $y = 5x + 1$, $(0,1)$ & $\left(-\frac{1}{5}, 0\right)$



Answers on the following page

Helpful Information

The **Cartesian plane** is a plane made up of an x-axis (a horizontal number line) and a y-axis (a vertical number line). An example of a Cartesian plane is on the right.



A **point** on the Cartesian Plane is given by a pair of integers called the **coordinates** of the point. A coordinate looks like this (x, y) where the first number is the x-coordinate (how far across) and the second number is the y-coordinate (how far up or down).

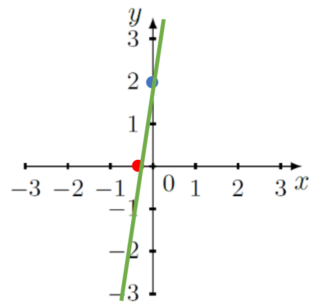
The **origin** is the intersection of the two axes and has coordinates (0,0).

When plotting a coordinate a helpful thought process is "crawl before you climb".

Example

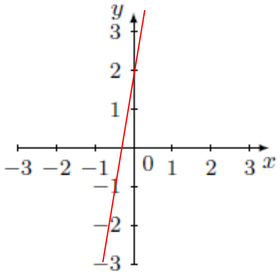
Question: The linear rule $y = 5x + 2$ has axis intercepts of $(0,2)$ and $\left(-\frac{2}{5}, 0\right)$. Use this information to sketch the linear graph.

Thought process and answer: The first coordinate (0,2) has an x-coordinate of zero so there is no horizontal move. The vertical move is 2 so we climb up 2. The second coordinate $\left(-\frac{2}{5}, 0\right)$ requires a horizontal move, or crawl, of $-\frac{2}{5}$. This fraction can be marked on the number line by first dividing the space between 0 and -1 into 5 equal parts. The vertical move is zero so this point remains on the x-axis. We then connect the points with a line to sketch the graph.

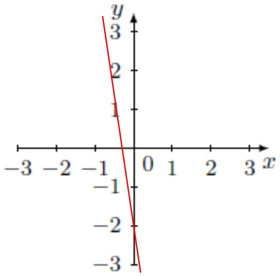


Answers

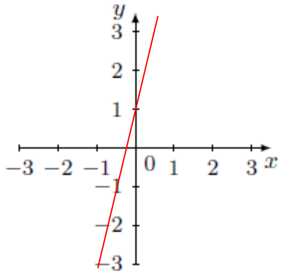
a. $y = 5x + 2$,
$(0,2)$ & $\left(-\frac{2}{5}, 0\right)$



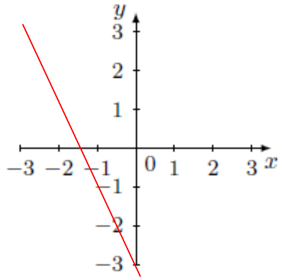
b. $y = -5x - 2$,
$(0,-2)$ & $\left(-\frac{2}{5}, 0\right)$



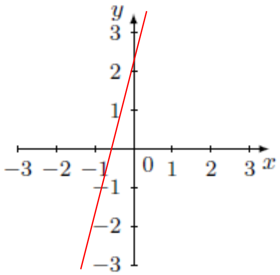
c. $y = 4x + 1$,
$(0,1)$ & $\left(-\frac{1}{4}, 0\right)$



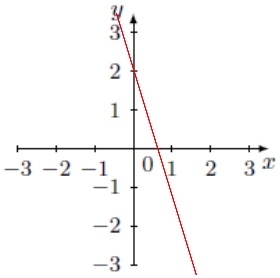
d. $y = 2x - 3$,
$(0,-3)$ & $\left(-\frac{3}{2}, 0\right)$



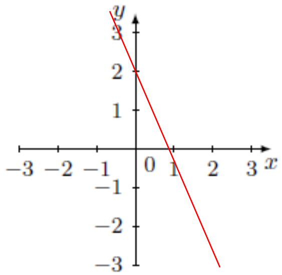
e. $y = 3x + 2$,
$(0,2)$ & $\left(-\frac{2}{3}, 0\right)$



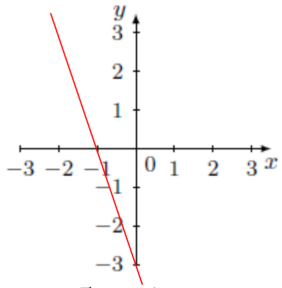
f. $y = -3x + 2$,
$(0,2)$ & $\left(\frac{2}{3}, 0\right)$



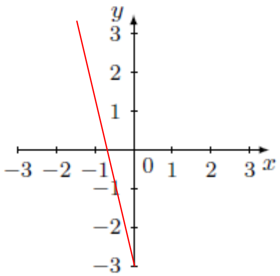
g. $y = -2x + 2$,
$(0,2)$ & $(1, 0)$



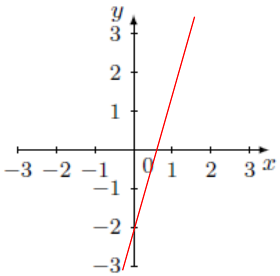
h. $y = -3x - 3$,
$(0,-3)$ & $(-1, 0)$



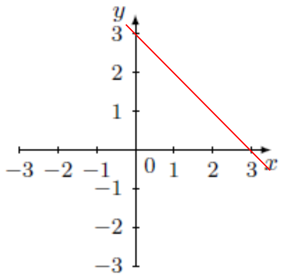
i. $y = -4x - 3$,
$(0,-3)$ & $\left(-\frac{3}{4}, 0\right)$



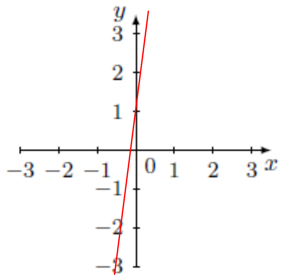
j. $y = 4x - 2$,
$(0,-2)$ & $\left(\frac{1}{2}, 0\right)$



k. $y = -x + 3$,
$(0,3)$ & $(3, 0)$



l. $y = 5x + 1$,
$(0,1)$ & $\left(-\frac{1}{5}, 0\right)$



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23 determining linear rules including negative gradients

Questions Part 1 of 2 – Determining a linear rule of the form $y = ax$ from a complete table of values

23.1 The points in each of the following tables follow a rule of the given form. Write the rule that connects the points within each table by filling in the box with the appropriate number.

a. $y = \square x$

x	-1	0	1	2	3	4
y	3	0	-3	-6	-9	-12

b. $y = \square x$

x	-1	0	1	2	3	4
y	-3	0	3	6	9	12

c. $y = \square x$

x	-1	0	1	2	3	4
y	5	0	-5	-10	-15	-20

d. $y = \square x$

x	-1	0	1	2	3	4
y	4	0	-4	-8	-12	-16

e. $y = \square x$

x	-1	0	1	2	3	4
y	-4	0	4	8	12	16

f. $y = \square x$

x	-1	0	1	2	3	4
y	7	0	-7	-14	-21	-28

g. $y = \square x$

x	-1	0	1	2	3	4
y	-9	0	9	18	27	36

h. $y = \square x$

x	-1	0	1	2	3	4
y	-5	0	5	10	15	20

Answers

a. $y = -3x$ b. $y = 3x$ c. $y = -5x$ d. $y = -4x$ e. $y = 4x$ f. $y = -7x$ g. $y = 9x$ h. $y = 5x$

Helpful Information

In a linear rule the number in front of the x tells us the "one-step size".

Think about the rule $y = -2x$ and its table of values:

x	-1	0	1	2	3	4
y	2	0	-2	-4	-6	-8

Every time the x-value increase by 1 we have one more group of -2 and so the y-value decreases by -2.

Example

Question: The points in the following table follow a rule of the form

$$y = \square x$$

x	-1	0	1	2	3	4
y	3	0	-3	-6	-9	-12

Write the rule that connects the points within the table by filling in the box with the appropriate number.

Thought process: The rule tells us that the y-value is a multiple of the x-value. Since each y-value is -3 times the x-value the number in the box must by -3.

Answer: $y = -3x$

Questions Part 2 of 2 – Determining a linear rule from a complete table of values

23.2 The points in each of the following tables follow a rule of the given form. Write the rule that connects the points within each table.

a.

x	-1	0	1	2	3	4
y		-6		-20		

b.

x	-1	0	1	2	3	4
y			-5		-11	

c.

x	-1	0	1	2	3	4
y		-5		9		

d.

x	-1	0	1	2	3	4
y		-5			7	

e.

x	-1	0	1	2	3	4
y	9				1	

f.

x	-1	0	1	2	3	4
y	9				-15	

g.

x	-1	0	1	2	3	4
y			0		10	

h.

x	-1	0	1	2	3	4
y			-7		-9	

i.

x	-1	0	1	2	3	4
y		2			-10	

j.

x	-1	0	1	2	3	4
y	5		19			

k.

x	-1	0	1	2	3	4
y	8			11		

l.

x	-1	0	1	2	3	4
y	14		0			

m.

x	-1	0	1	2	3	4
y			-3		-5	

n.

x	-1	0	1	2	3	4
y	1				-23	

Answers

a. $y = -7x - 6$ b. $y = -3x + 2$ c. $y = 7x - 5$ d. $y = 4x - 5$ e. $y = -2x + 7$ f. $y = -6x + 3$ g. $y = 5x - 5$ h. $y = -x - 6$ i. $y = -4x + 2$ j. $y = 7x + 12$ k. $y = x + 9$ l. $y = -7x + 7$ m. $y = -x - 2$ n. $y = -6x - 5$

Helpful Information

Thinking about a Linear Rule

If a collection of coordinates satisfy a linear rule then the one-step size is constant. Note: The one-step size is the amount that y-value changes by each time the x-value increases by 1.

How to Find a Linear Rule

1. Write the general rule $y = mx + c$
2. Determine the one-step size
   - Calculate the difference between the y-values in the given coordinates
   - Divide this difference by the number of steps between the coordinates
   - If the values are increasing the one-step size is positive, if the values are decreasing the one-step size is negative
3. Replace the m in the general rule with the value of the one step size
4. Substitute a coordinate into the general rule using a pair of x and y values from the table
5. Solve the equation in step 4 for c
6. Complete the rule by filling in the value of m and c in the table

Example

Question: Write the rule that connects the points in the table.

x	-1	0	1	2	3	4
y		-6		-20		

Thought process: Using the above process we have…

1. $y = mx + c$
2. The difference between the y-values is 14.
   There are 2 steps between the coordinates so divide 14 by 2
   The values are decreasing so the one-step size is -7
3. $y = -7x + c$
4. Substituting the coordinate (0,-6) into the rule from step 3 gives:
   $$-6 = -7 \times 0 + c$$
5. Solving this equation gives $c = -6$
6. The rule is $y = -7x - 6$

Answer: $y = -7x - 6$

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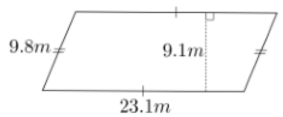
24 choosing and using formulas in measurement

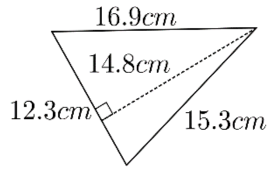
Questions Part 1 of 3 – Perimeter of parallelograms and triangles

24.1 To the nearest whole number, what is the perimeter of each shape below? You may use your calculator if you wish.

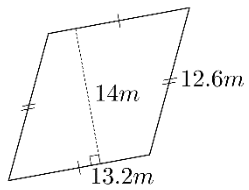
a.



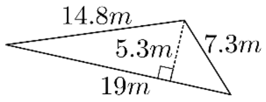
b.



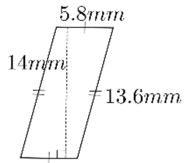
c.



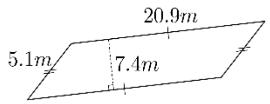
d.



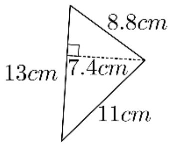
e.



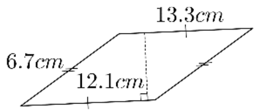
f.



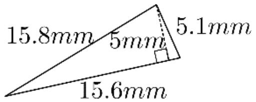
g.



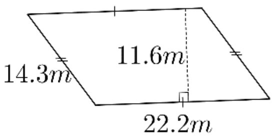
h.



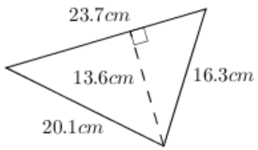
i.



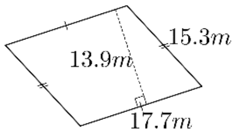
j.



k.



l.



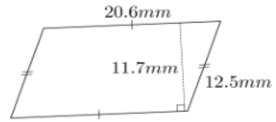
Answers

a. 66mm b. 45cm c. 52m d. 41m e. 39mm f. 52m g. 33cm h. 40cm i. 37mm j. 73m k. 60cm l. 66m

Helpful Information

The **perimeter** of an object is the total length around its outside

Note that if a shape has little dashes on its sides this is used to show that some of the sides have equal lengths.
For example, in the shape below it looks like only 2 of the 4 lengths around the outside are given (20.6mm and 12.5mm)



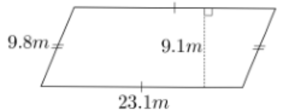
However, since the bottom length has one dash, just like the top, we know it must be 20.6mm as well. Similarly, since the left side has two dashes, just like the right side, we know it must be 12.5mm.

Strategy for finding the perimeter to the nearest whole number

1. Add up the lengths around the outside
2. Round to the nearest whole number
3. Include appropriate units

Example

Question: To the nearest whole number, what is the perimeter of the shape below?



Thought process: Using the above strategy we have…

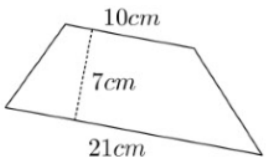
1. 23.1+9.8+23.1+9.8=65.8 (as the top side is 23.1m and the right side is 9.8m)
2. Rounding 65.8 to the nearest whole number gives 66.
3. The perimeter is 66m

Answer: 66m

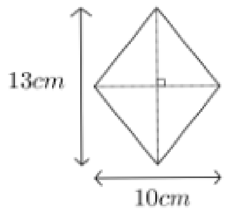
Questions Part 2 of 3 – Area of parallelograms, triangles, kites and trapeziums

24.2 Calculate the area of the shapes below, give your answers to the nearest whole number. You may use your calculator if you wish.

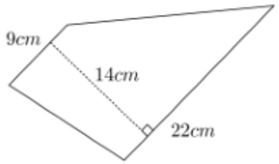
a.



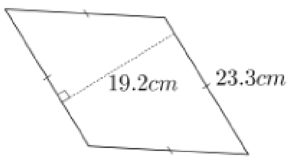
b.



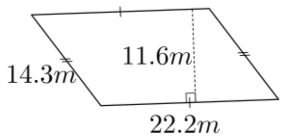
c.



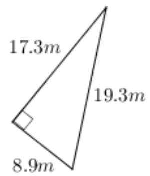
d.



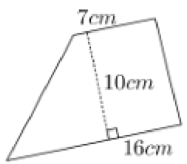
e.



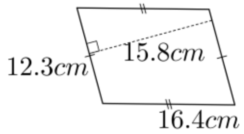
f.



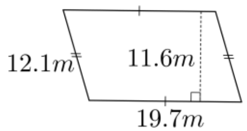
g.



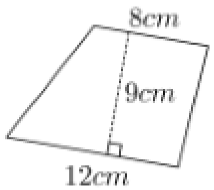
h.



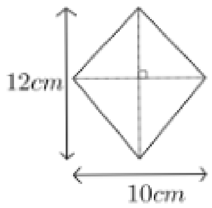
i.



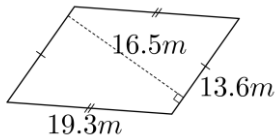
j.



k.



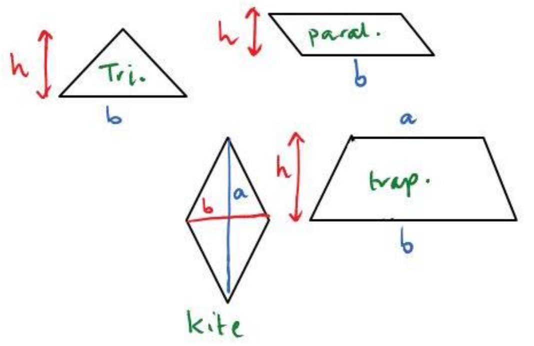
l.



Answers

a. 109cm^2 b. 65cm^2 c. 217cm^2 d. 447cm^2 e. 258m^2 f. 77m^2 g. 115cm^2 h. 194cm^2 i. 229m^2 j. 90cm^2 k. 60cm^2 l. 224m^2

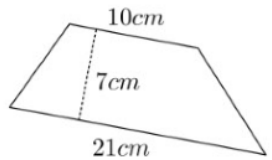
Helpful Information

Area Formulas	
Area of a Triangle: $A = \frac{1}{2} \times b \times h$ Area of a Parallelogram: $A = b \times h$ Area of a Kite: $A = \frac{1}{2} \times a \times b$ Area of a Trapezium: $A = \frac{1}{2}(a + b) \times h$	***Base and height must meet at right angles***

Strategy for choosing the right formula and using it to calculate a measurement

1. Identify the type of shape you've been given
2. Choose the appropriate formula
3. Substitute the values for each pronumeral using the diagram
4. Evaluate the expression
5. Round to the nearest whole number
6. Include appropriate units

Example



Question: Calculate the area of the shape on the right, give your answer to the nearest whole number.

Thought process: Using the above strategy we have…

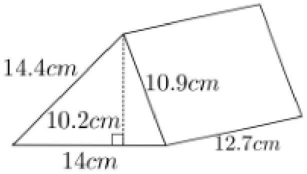
1. The shape is a trapezium
2. The formula for the area of a trapezium is $A = \frac{1}{2}(a + b) \times h$
3. For the given trapezium $a = 10$, $b = 21$ and $h = 7$ and so
$$A = \frac{1}{2}(10 + 21) \times 7$$
4. Evaluating $A = \frac{1}{2}(10 + 21) \times 7$ gives 108.5
5. Rounding 108.5 to the nearest whole number gives 109
6. 109cm^2

Answer: 109cm^2

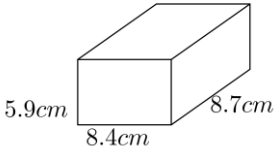
Questions Part 3 of 3 – Volume of Prisms

24.3 Calculate the volume of the shapes below, give your answers to the nearest whole number. You may use your calculator if you wish.

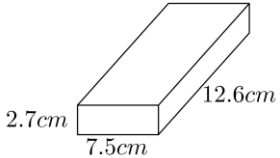
a.



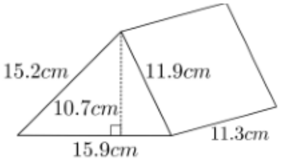
b.



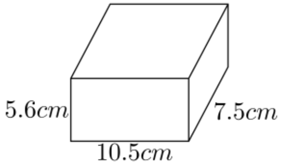
c.



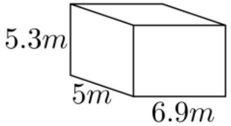
d.



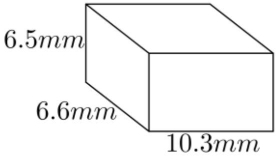
e.



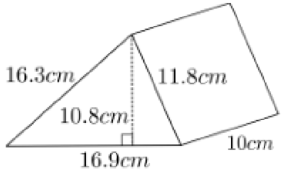
f.



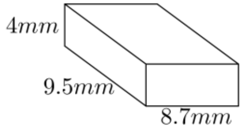
g.



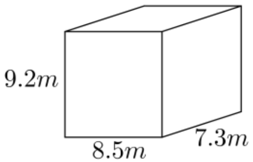
h.



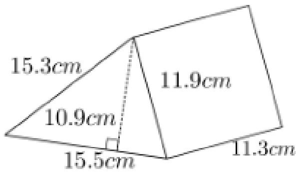
i.



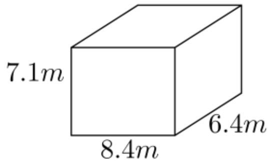
j.



k.



l.



Answers

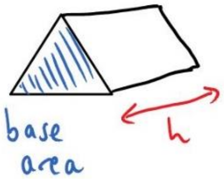
a. 907cm³ b. 431cm³ c. 255cm³ d. 961cm³ e. 441cm³ f. 183m³ g. 442mm³ h. 913cm³
i. 331mm³ j. 571m³ k. 955cm³ l. 382m³

Helpful Information

A prism is a shape where each cross section is the same shape. The shape of the cross section is referred to as the "base".

Volume of a prism =
$base\ area\ \times height$

The "height" of a prism is how far back the "base area" goes

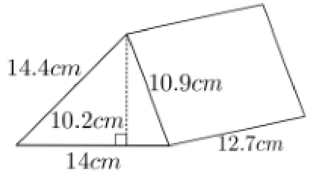


Strategy for choosing the right formula and using it to calculate a measurement

1. Identify the type of shape you've been given
2. Choose the appropriate formula
3. Substitute the values for each pronumeral using the diagram
4. Evaluate the expression
5. Round to the nearest whole number
6. Include appropriate units

Example

Question: Calculate the volume of the shape on the right, give your answer to the nearest whole number.



Thought process: Using the above strategy we have…

1. The shape is a (triangular) prism
2. The formula for the volume of a prism is $base\ area\ \times height$
3. For the given prism, the "base" is a triangle and so its area is calculated with the formula $A = \frac{1}{2} \times b \times h$. So we calculate the base area as $\frac{1}{2} \times 14 \times 10.2$. The triangle goes back 12.7cm so the "height" of the prism is 12.7. The volume is then calculated with the expression $\frac{1}{2} \times 14 \times 10.2 \times 12.7$
4. Evaluating $\frac{1}{2} \times 14 \times 10.2 \times 12.7$ gives 906.78
5. Rounding 906.78 to the nearest whole number gives 907
6. 907cm³

Answer: 907cm³

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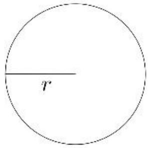
25 features of a circle

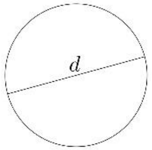
Questions Part 1 of 3 – Radius and diameter of a circle

25.1 Calculate the requested feature of each circle.

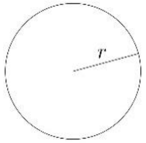
a. Radius = 18.3cm, calculate the diameter



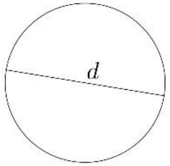
b. Diameter = 41.2cm, calculate the radius



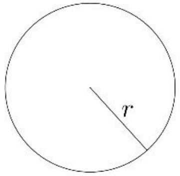
c. Radius = 6.7cm, calculate the diameter



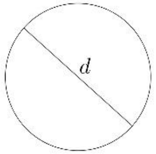
d. Diameter = 64.2cm, calculate the radius



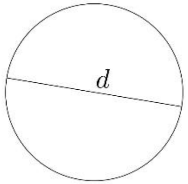
e. Radius = 28.9cm, calculate the diameter



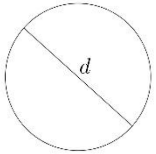
f. Diameter = 49.8cm, calculate the radius



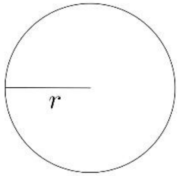
g. Diameter = 31.4cm, calculate the radius



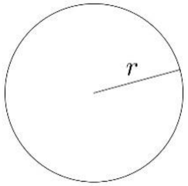
h. Diameter = 35.6cm, calculate the radius



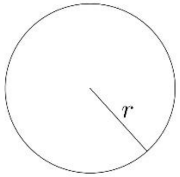
i. Radius = 15.4cm, calculate the diameter



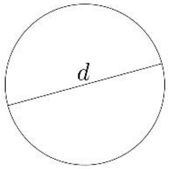
j. Radius = 14.8cm, calculate the diameter



k. Radius = 13.6cm, calculate the diameter



l. Diameter = 18.8cm, calculate the radius



Answers

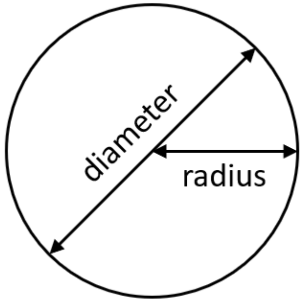
a. d = 36.6cm b. r = 20.6cm c. d = 13.4cm d. r = 32.1cm e. d = 57.8cm f. r = 24.9cm
g. r = 15.7cm h. r = 17.8cm i. d = 30.8cm j. d = 29.6cm k. d = 27.2cm l. r = 9.4cm

Helpful Information

The **radius** of a circle is a straight line from the centre to the outside edge

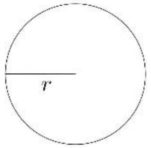
The **diameter** of a circle a straight line from side to side that passes through the centre

$$diameter = 2 \times radius$$
$$radius = diameter \div 2$$



Examples

Question: The circle below has a radius of 18.3cm. What is its diameter?



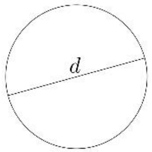
Thought process: The diameter is twice as long as the radius and so

$$d = 18.3 \times 2 = 36.6$$

The units for both measurements is cm and therefore the diameter is 36.6cm.

Answer: d = 36.6cm

Question: The circle below has a diameter of 41.2cm. What is its radius?



Thought process: The radius is half the length of the diameter and so

$$r = 41.2 \div 2 = 20.6$$

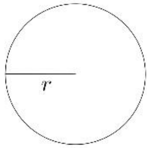
The units for both measurements is cm and therefore the radius is 20.6cm.

Answer: r = 20.6cm

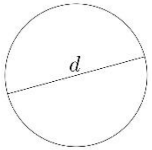
Questions Part 2 of 3 – Circumference of a circle

25.2 Calculate the circumference of each circle, giving your answers to 2dp.

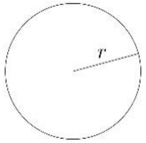
a. Radius = 18.3cm, calculate the circumference



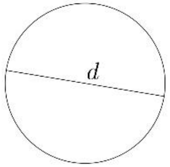
b. Diameter = 41.2cm, calculate the circumference



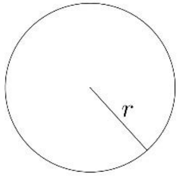
c. Radius = 6.7cm, calculate the circumference



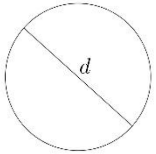
d. Diameter = 64.2cm, calculate the circumference



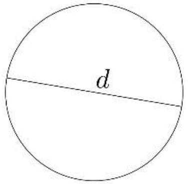
e. Radius = 28.9cm, calculate the circumference



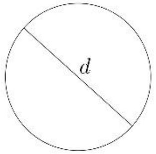
f. Diameter = 49.8cm, calculate the circumference



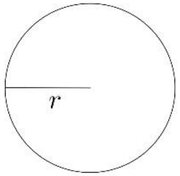
g. Diameter = 31.4cm, calculate the circumference



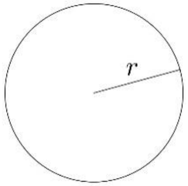
h. Diameter = 35.6cm, calculate the circumference



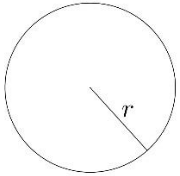
i. Radius = 15.4cm, calculate the circumference



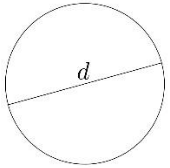
j. Radius = 14.8cm, calculate the circumference



k. Radius = 13.6cm, calculate the circumference



l. Diameter = 18.8cm, calculate the circumference

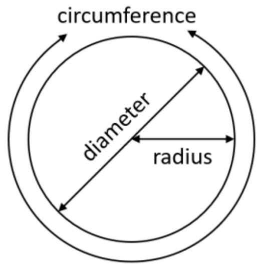


Answers

a. c = 114.98cm b. c = 129.43cm c. c = 42.10cm d. c = 201.69cm e. c = 181.58cm f. c = 156.45cm g. c = 98.65cm h. c = 111.84cm i. c = 96.76cm j. c = 92.99cm k. c = 85.45cm l. c = 59.06cm

Helpful Information

The **circumference** of a circle is the length around the outside.



The circumference is made up of just over 3 lengths of the diameter. The actual number of times the diameter fits around the circle is represented by π, called "pi", which has infinitely many decimal places...!

Written to the first 6 decimal places we have $\pi \approx 3.141592$

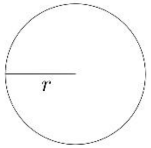
$$circumference = \pi \times d$$
$$= 2 \times \pi \times r$$

One way to remember these numbers is to think "I wish I could calculate pi" as the number of letters in each word gives the first six decimal places 😊

To help remember the meanings of radius, diameter and circumference put the words in order from smallest number of letters to most number of letters (as they are ordered in this sentence) as the shortest word (radius) is the shortest length and the longest word (circumference) is the longest length.

Examples

Question: This circle has a radius of 18.3cm. What is its circumference? Give your answer to 2dp.



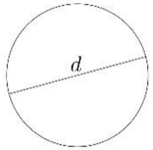
Thought process: The formula relating circumference to radius is:

$$c = 2 \times \pi \times r$$
$$\approx 2 \times 3.141592 \times 18.3$$
$$\approx 114.982267$$

Rounding to 2dp. and included units we have c = 114.98cm

Answer: c = 114.98cm

Question: This circle has a diameter of 41.2cm. What is its circumference? Give your answer to 2dp.



Thought process: The formula relating circumference to diameter is:

$$c = \pi \times d$$
$$\approx 3.141592 \times 41.2$$
$$\approx 129.433590$$

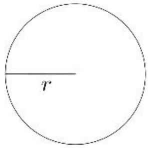
Rounding to 2dp. and included units we have c = 129.43cm

Answer: c = 129.43cm

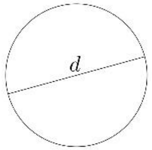
Questions Part 3 of 3 – Area of a circle

25.3 Calculate the area of each circle, giving your answers to 2dp.

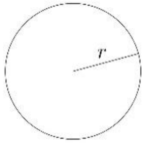
a. Radius = 18.3cm, calculate the area



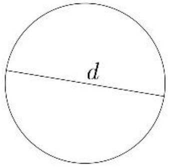
b. Diameter = 41.2cm, calculate the area



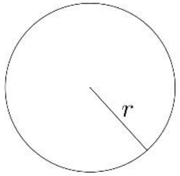
c. Radius = 6.7cm, calculate the area



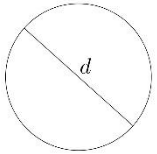
d. Diameter = 64.2cm, calculate the area



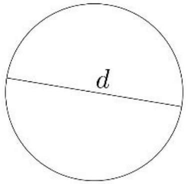
e. Radius = 28.9cm, calculate the area



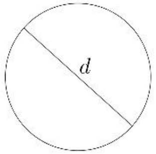
f. Diameter = 49.8cm, calculate the area



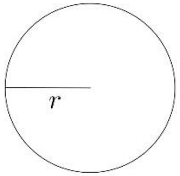
g. Diameter = 31.4cm, calculate the area



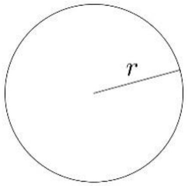
h. Diameter = 35.6cm, calculate the area



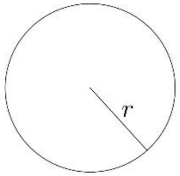
i. Radius = 15.4cm, calculate the area



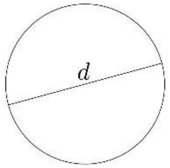
j. Radius = 14.8cm, calculate the area



k. Radius = 13.6cm, calculate the area



l. Diameter = 18.8cm, calculate the area



Answers

a. a = 1052.09cm^2 b. a = 1333.17cm^2 c. a = 141.03cm^2 d. a = 3237.13cm^2
e. a = 2623.89cm^2 f. a = 1947.82cm^2 g. a = 774.37cm^2 h. a = 995.38cm^2 i. a = 745.06cm^2
j. a = 688.13cm^2 k. a = 581.07cm^2 l. a = 277.59cm^2

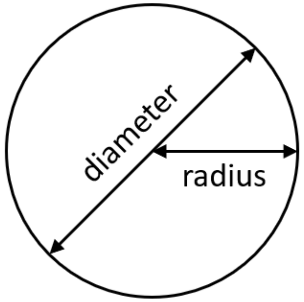
Helpful Information

The area of a circle is calculated using the formula below:

$$A = \pi \times r^2$$

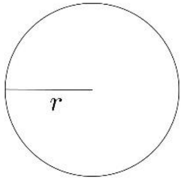
Recall that π, correct to 6 decimal places is:

$$\pi \approx 3.141592$$



Examples

Question: The circle on the right has a radius of 18.3cm. What is its area?



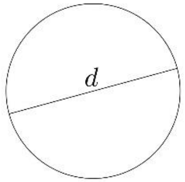
Thought process: The formula relating the area of a circle to its radius is:

$$A = \pi \times r^2$$
$$\approx 3.141592 \times 18.3^2$$
$$\approx 1052.087745$$

Rounding to 2dp. and included units we have A = 1052.09cm^2

Answer: A = 1052.09cm^2

Question: The circle on the right has a diameter of 41.2cm. What is its area?



Thought process: As the formula for the area of a circle requires the radius, we first need to determine the radius of the circle. Recall that the radius is half the diameter and so we have.

$$r = 41.2 \div 2 = 20.6$$

The formula relating the area of a circle to its radius is:

$$A = \pi \times r^2$$
$$\approx 3.141592 \times 20.6^2$$
$$\approx 1333.165981$$

Rounding to 2dp. and included units we have A = 1333.17cm^2

Answer: A = 1333.17cm^2

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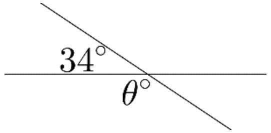
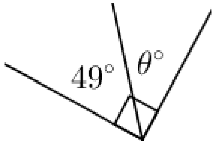
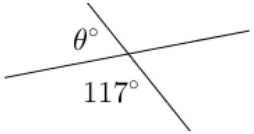
26 angles around a point and around parallel lines

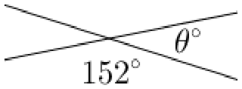
Questions Part 1 of 6 – Angles around a point

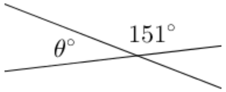
26.1 In each diagram **c**lassify the relationship between the two marked angles as complementary, supplementary or vertically opposite, then find the value of θ.

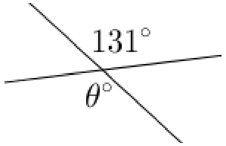
a.


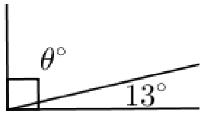
b.


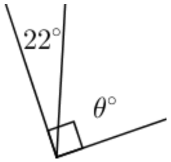
c.


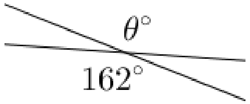
d.


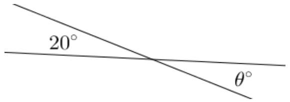
e.


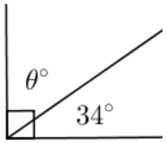
f.


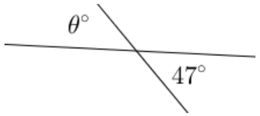
g.


h.


i.


j.


k.


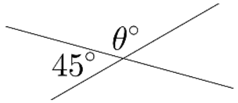
l.


Answers

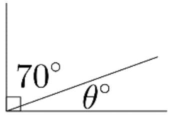
a. supplementary, θ=146 b. complementary, θ=41 c. supplementary, θ=63
d. supplementary, θ=28 e. supplementary, θ=29 f. vertically opposite, θ=131
g. complementary, θ=77 h. complementary, θ=68 i. vertically opposite, θ=162
j. vertically opposite, θ=20 k. complementary, θ=56 l. vertically opposite, θ=47

Helpful Information

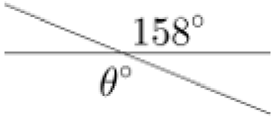
Two angles that add to 180° are called **supplementary angles**. This means that supplementary angles will form a straight line. In the diagram on the right the marked angles, 45° and θ° are **supplementary angles**.



Two angles that add to 90° are called **complementary angles**. This means that complementary angles will form a right angle.
In the diagram on the right the marked angles, 70° and θ° are **complementary angles**.



When two lines intersect four angles are formed. The angles opposite each other are called **vertically opposite**. These angles are always equal (the same size). In the diagram on the right the marked angles, 158° and θ° are **vertically opposite angles**.



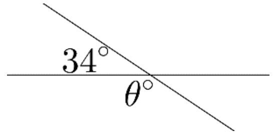
The following (somewhat silly) sentences may help you remember when to use complementary and when to use supplementary:

You will get a compliment if you are right (complementary – right)
What's sup bro? Talk straight with me! (supplementary – straight)

The symbol θ in each diagram is called theta and is a Greek letter. In the same way that x is often used to represent an unknown number, Greek letters are often used to represent an unknown angle.

Example

Question: Classify the relationship between the two marked angles as complementary, supplementary or vertically opposite, then find the value of θ



Thought process: Since the angles form a straight line they are supplementary. This means that the angles add to 180. $34 + \theta = 180$, so $\theta = 180 - 34 = 146$
Answer: supplementary, θ**=146**

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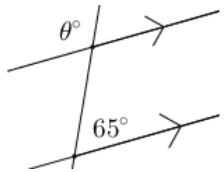
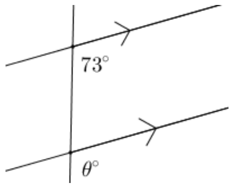
Questions Part 2 of 6 – Identifying corresponding angles

26.2 Determine which of the following pairs of angles are corresponding.

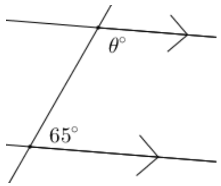
a.



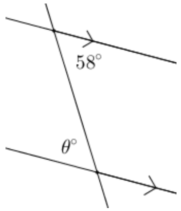
b.



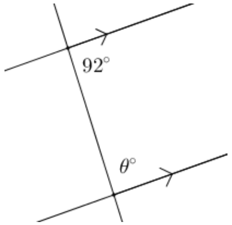
c.



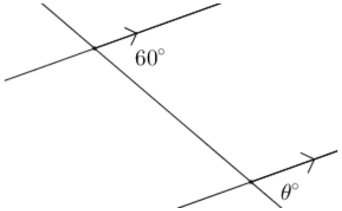
d.



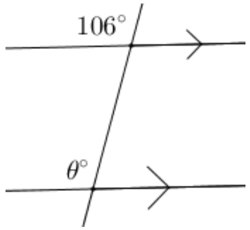
e.



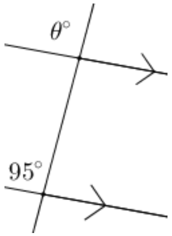
f.



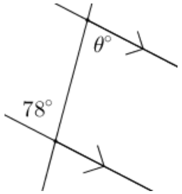
g.



h.



i.



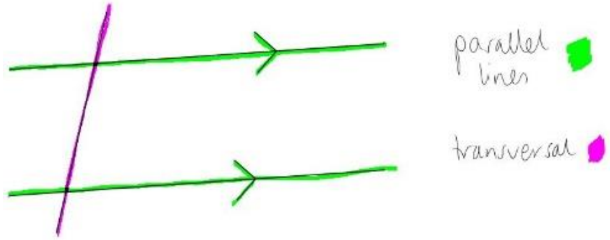
Answers

a. not corresponding b. corresponding c. not corresponding d. not corresponding e. not corresponding f. corresponding g. corresponding h. corresponding i. not corresponding

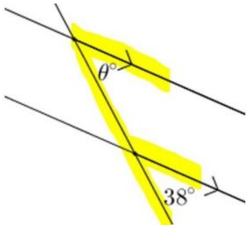
Helpful Information

Two lines are called **parallel** if they will never meet no matter how far they are extended in either direction. Note that arrows are used to show that two lines are parallel.

A **transversal** is the name given to a line that crosses two other lines.



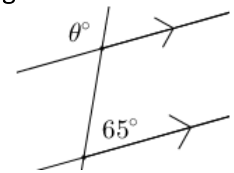
There are three key classifications of certain pairs of angles involving two lines and a transversal; corresponding, co-interior and alternate. In this subskill we will focus on corresponding angles.



Corresponding angles are in corresponding positions. In the diagram on the right the marked angles, 38° and $\theta°$ are **corresponding angles**. We say that the angles are in corresponding positions because both are "bottom right" of the intersection point. A good check to make sure you have corresponding angles is to check if they are contained within an F. The 'F' is shown on the diagram on the right. Note however that the F may look like or even

Example

Question: Are the pairs of angles marked below corresponding?



Thought process: The angle marked $\theta°$ is top left and the angle marked 65° is top right so these angles are not in corresponding positions.

Answer: No

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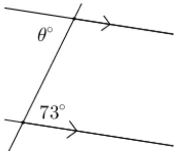
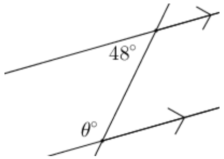
Questions Part 3 of 6 – identifying co-interior angles

26.3 Determine which of the following pairs of angles are co-interior.

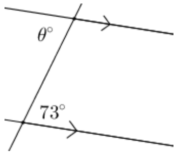
a.



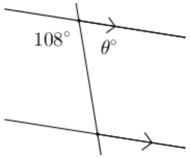
b.



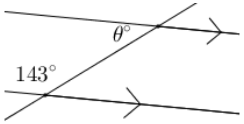
c.



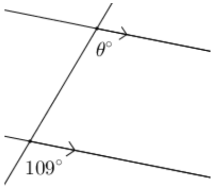
d.



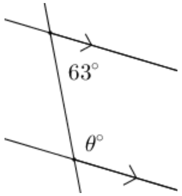
e.



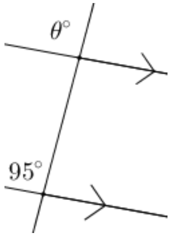
f.



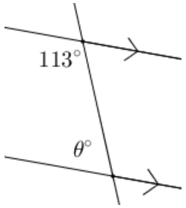
g.



h.



i.

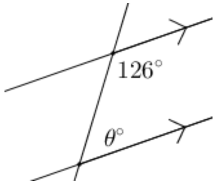


Answers

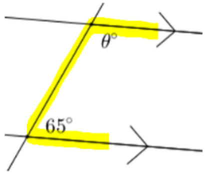
a. not co-interior b. co-interior c. not co-interior d. not co-interior e. co-interior
f. not co-interior g. co-interior h. not co-interior i. co-interior

Helpful Information

Co-interior angles are just inside each of the parallel lines and on the same side of the transversal. In the diagram on the right the marked angles, 126° and $\theta°$ are **co-interior angles**.

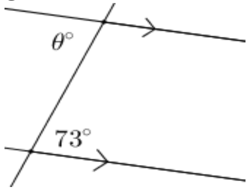


While co-interior angles are generally easy to spot, seeing if they are contained within a c is a good check. The 'c' is shown on the diagram on the right. Note however that the c may look like ɔ or even ɔ



Example

Question: Are the pairs of angles marked below co-interior?



Thought process: Both angles are just inside each of the parallel lines, though they are not on the same side of the transversal so these angles are not co-interior.

Answer: No

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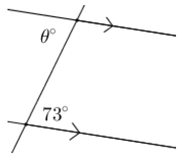
Questions Part 4 of 6 – Identifying alternate angles	*Helpful Information*

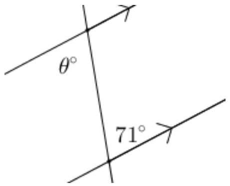
Questions Part 4 of 6 – Identifying alternate angles

26.4 Determine which of the following pairs of angles are alternate.

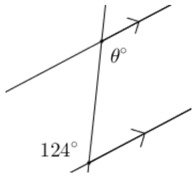
a.



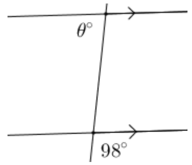
b.



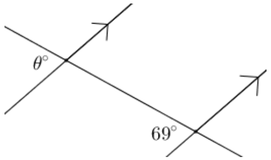
c.



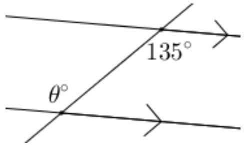
d.



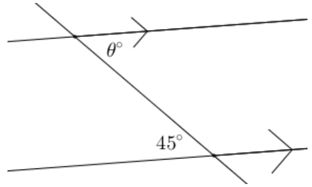
e.



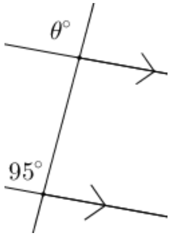
f.



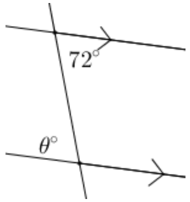
g.



h.



i.

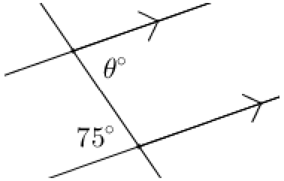


Answers

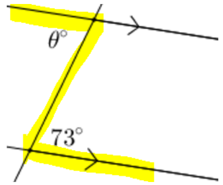
a. alternate b. alternate c. alternate d. not alternate e. not alternate f. alternate
g. alternate h. not alternate i. alternate

Helpful Information

Alternate angles are just inside each of the parallel lines and on alternate (different) sides of the transversal. In the diagram on the right the marked angles, 75° and $\theta°$ are **alternate angles**.

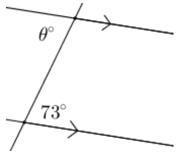


While alternate angles can be easy to spot, seeing if they are held within the letter z is a good check. The 'z' is shown on the diagram on the right. The z may look like or even



Example

Question: Are the pairs of angles marked below alternate?



Thought process: Both angles are just inside each of the parallel lines and are on alternate sides of the transversal so these angles are alternate.

Answer: Yes

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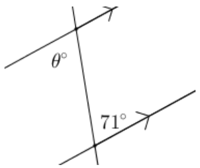
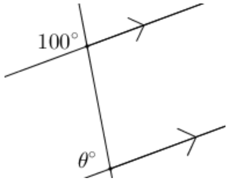
Questions Part 5 of 6 – Classifying angles as corresponding, co-interior and alternate

26.5 Classify the relationship between the two marked angles as co-interior, corresponding or alternate.

a.



b.



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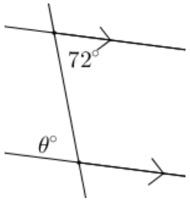

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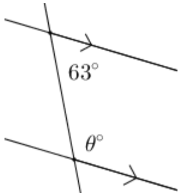

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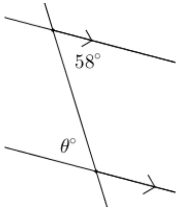

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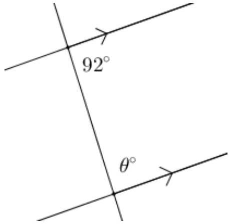

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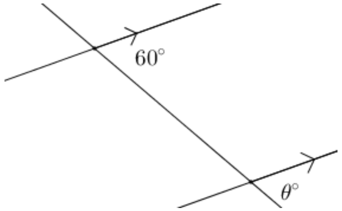

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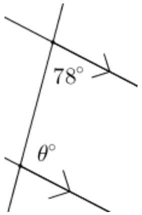

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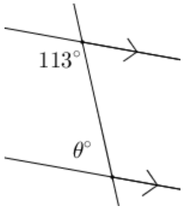

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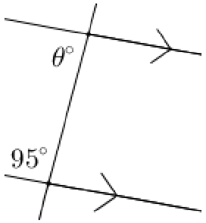
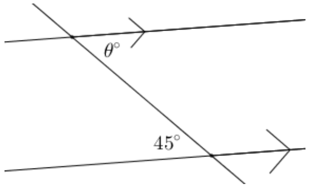
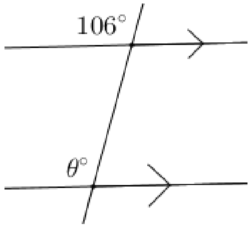


k.



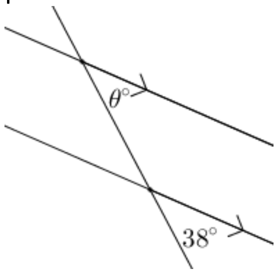
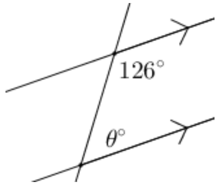
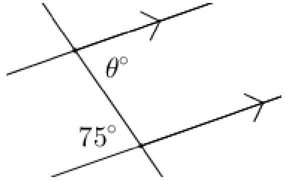
l.



Answers

a. alternate b. corresponding c. alternate d. co-interior e. alternate f. co-interior
g. corresponding h. co-interior i. co-interior j. co-interior k. alternate l. corresponding

Helpful Information

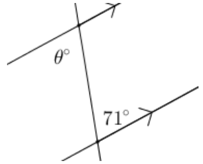
Corresponding angles are in corresponding positions 	Co-interior angles are just inside each of the parallel lines on the same side of the transversal 	Alternate angles are just inside each of the parallel lines and on alternate sides of the transversal 

Strategy for classifying angles as corresponding, co-interior or alternate

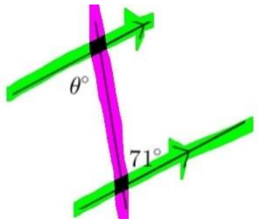
1. Mark the two parallel lines in one colour
2. Mark the transversal in another colour
3. If
   a. The two marked angles are both just inside each parallel line then look to see if
      i. They are on the same side of the transversal (co-interior)
      ii. They are on alternate sides of the transversal (alternate)
   b. If they are not both just inside the parallel lines look to see if they are in corresponding positions (corresponding)

Example

Question: Classify the relationship between the two marked angles as co-interior, corresponding or alternate.



Thought process: Using the above strategy we have…
Both angles are just inside each parallel line, and they are on alternate sides of the transversal.
Answer: Alternate



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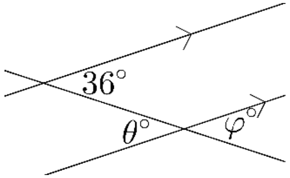
Questions Part 6 of 6 – Classifying and determining unknown angles around parallel lines

26.6 In each diagram,

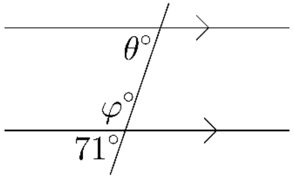
i. determine the values of θ and φ and,

ii. state the relationship between θ and φ

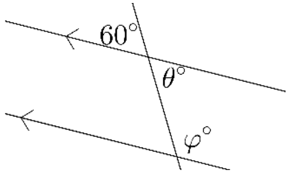
a.



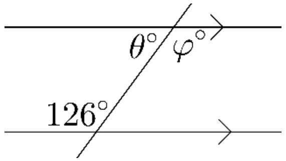
b.



c.



d.



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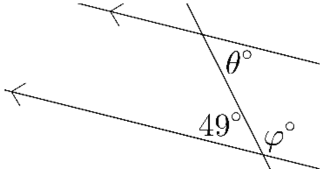

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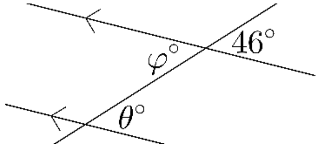
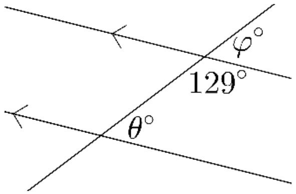


g.



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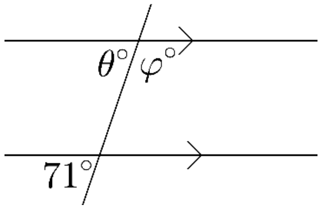
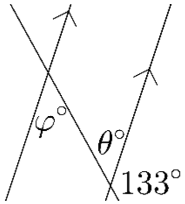
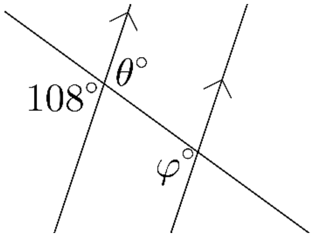


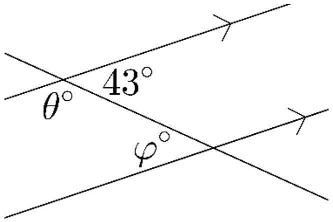
i.



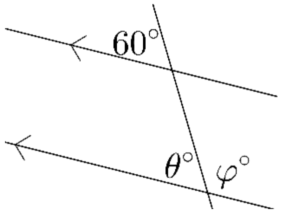
j.



k.



l.



Answers

a.i. $\theta = 36, \varphi = 36$ ii. vertically opposite b.i. $\theta = 71, \varphi = 109$ ii. co-interior

c.i. $\theta = 60, \varphi = 120$ ii. co-interior d.i. $\theta = 54, \varphi = 126$ ii. supplementary

e.i. $\theta = 49, \varphi = 131$ ii. co-interior f.i. $\theta = 46, \varphi = 46$ ii. alternate

g.i. $\theta = 51, \varphi = 51$ ii. corresponding h.i. $\theta = 71, \varphi = 109$ ii. supplementary

i.i. $\theta = 47, \varphi = 47$ ii. alternate j.i. $\theta = 108, \varphi = 108$ ii. alternate

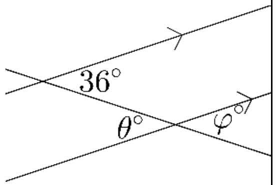
k.i. $\theta = 137, \varphi = 43$ ii. co-interior l.i. $\theta = 60, \varphi = 120$ ii. supplementary

Helpful Information

- **Supplementary** angles add to 180°
- **Vertically opposite** angles are equal in value
- **Corresponding** angles are equal in value
- **Alternate** angles are equal in value
- **Co-interior** angles add to 180°

Examples

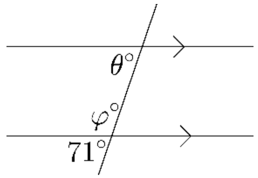
Question: For the diagram on the right,
i. determine the values of θ and φ and,
ii. state the relationship between θ and φ



Thought process: The angle marked 36° and θ are alternate angles (contained within a z) and are equal. So $\theta = 36$. θ and φ are vertically opposite angles (contained within an x) and are equal in value. So $\varphi = 36$

Answer: i. $\theta = 36$ and $\varphi = 36$. ii. θ and φ are vertically opposite.

Question: For the diagram on the right,
i. determine the values of θ and φ and,
ii. state the relationship between θ and φ



Thought process: The angle marked 71° and θ are corresponding angles (contained within an F) and are equal. So $\theta = 71$. The angle marked 71° and φ are supplementary (on a straight line) and add t0 180°. So $\varphi = 180 - 71 = 109$. θ and φ are co-interior angles (contained with in a c).

Answer: i. $\theta = 71$ and $\varphi = 109$. ii. θ and φ are co-interior angles.

27 probabilities from two-way tables

27.1 Calculate the following probabilities using the information given in the two-way tables below. Remember to give fractional answers in simplified form.

a.

	C	C'	Total
D	4	4	8
D'	13	6	19
Total	17	10	27

i. P(C)
ii. P(C and D)

b.

	C	C'	Total
D	7	13	20
D'	15	2	17
Total	22	15	37

i. P(D)
ii. P(C and D)

c.

	B	B'	Total
D	9	14	23
D'	2	12	14
Total	11	26	37

i. P(B)
ii. P(not B)

d.

	A	A'	Total
C	4	14	18
C'	9	11	20
Total	13	25	38

i. P(A)
ii. P(A and C)

e.

	B	B'	Total
C	13	14	27
C'	14	4	18
Total	27	18	45

i. P(C)
ii. P(not B)

f.

	A	A'	Total
C	11	12	23
C'	14	4	18
Total	25	16	41

i. P(C)
ii. P(A and C)

g.

	B	B'	Total
C	9	14	23
C'	8	3	11
Total	17	17	34

i. P(B)
ii. P(not C)

h.

	C	C'	Total
D	14	13	27
D'	11	15	26
Total	25	28	53

i. P(D)
ii. P(not D)

i.

	B	B'	Total
D	14	3	17
D'	6	8	14
Total	20	11	31

i. P(B)
ii. P(not D)

j.

	B	B'	Total
C	6	10	16
C'	6	9	15
Total	12	19	31

i. P(C)
ii. P(B and C)

k.

	A	A'	Total
C	6	13	19
C'	5	13	18
Total	11	26	37

i. P(A)
ii. P(not C)

l.

	A	A'	Total
C	1	3	4
C'	15	11	26
Total	16	14	30

i. P(C)
ii. P(A and C)

Answers

a.i. 17/27 ii. 4/27 b.i. 20/37 ii. 7/37 c.i. 11/37 ii. 26/37 d.i. 13/38 ii. 2/19 e.i. 3/5 ii. 2/5 f.i. 23/41 ii. 11/41 g.i. 1/2 ii. 11/34 h.i. 27/53 ii. 26/53 i.i. 20/31 ii. 14/31 j.i. 16/31 ii. 6/31 k.i. 11/37 ii. 18/37 l.i. 2/15 ii. 1/30

Helpful Information

A two-way table represents information collected about two different categories.

Imagine for example that a group of students were asked two questions: 1. Do you have a dog? and 2. Do you enjoy eating apples?

Students either have a dog (represented by D) or don't have a dog (represented by D'). Similarly, students either like apples (represented by A) or don't like apples (represented by A').

A possible two-way table that could be created from asking 37 students this question is shown on the right. Here we have

- 12 students who have a dog and enjoy eating apples,
- 5 students who have dog but don't enjoy eating apples,
- 6 students who don't have a dog but enjoy eating apples and
- 14 students who don't have a dog and don't like eating apples.

	A	A'	Total
D	12	5	17
D'	6	14	20
Total	18	19	37

We can calculate probabilities from two-way tables. Recall that a **probability** is calculated using

$$P(\text{event}) = \frac{\text{the number of favourable outcomes}}{\text{the total number of outcomes}}$$

Example

Question: Calculate the following probabilities using the information in the two-way table on the right.
i. P(C)
ii. P(C and D)

	C	C'	Total
D	4	4	8
D'	13	6	19
Total	17	10	27

Thought process: The number of outcomes for C is 17. The total number of outcomes is 27. So P(C)=17/27. The number of outcomes for C and D is 4. So P(C and D)=4/27

Answer: i. P(C)=17/27 ii. P(C and D)=4/27

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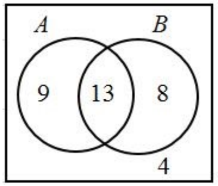
28 probabilities from venn diagrams

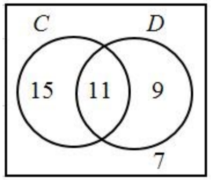
28.1 Calculate the following probabilities using the information given in the two-way tables below. Remember to give fractional answers in simplified form.

a.



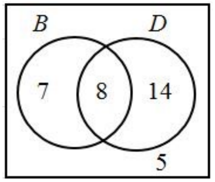
i. P(A)
ii. P(A or B or both)

b.



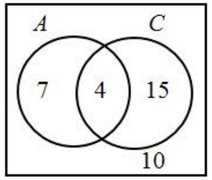
i. P(D)
ii. P(C and D)

c.



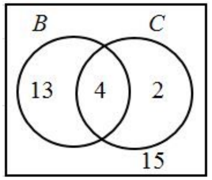
i. P(D)
ii. P(not B)

d.



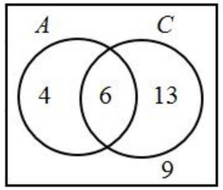
i. P(A)
ii. P(A and C)

e.



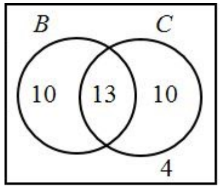
i. P(B)
ii. P(not C)

f.



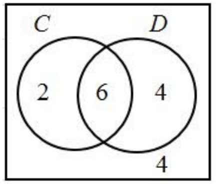
i. P(A)
ii. P(A or C or both)

g.



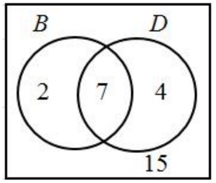
i. P(C)
ii. P(not B)

h.



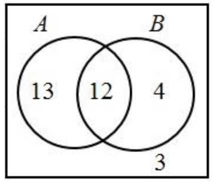
i. P(C)
ii. P(C and D)

i.



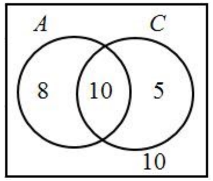
i. P(D)
ii. P(B and D)

j.



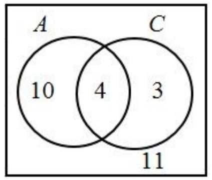
i. P(A)
ii. P(A or B or both)

k.



i. P(C)
ii. P(not C)

l.

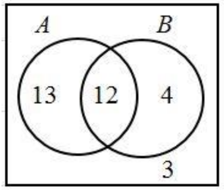


i. P(A)
ii. P(not C)

Answers
a.i. 11/17 ii. 15/17 b.i. 10/21 ii. 11/42 c.i. 11/17 ii. 19/34 d.i. 11/36 ii. 1/9 e.i. 1/2 ii. 14/17
f.i. 5/16 ii. 23/32 g.i. 23/37 ii. 14/37 h.i. 1/2 ii. 3/8 i.i. 11/28 ii. 1/4 j.i. 25/32 ii. 29/32
k.i. 15/33 ii. 18/33 l.i. 1/2 ii. 3/4

Helpful Information
A Venn diagram represents information collected about two different categories.

Imagine for example that a group of students were asked two questions: 1. Do you enjoy eating apples? 2. Do you enjoy eating blueberries?



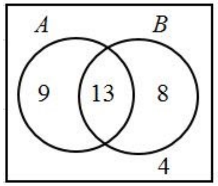
A possible Venn diagram that could be created from asking 32 students this question is shown on the right. Here we have

- 13 students who enjoy eating apples and don't like eating blueberries,
- 12 students who enjoy eating apples and blueberries,
- 4 students who don't enjoy eating apples and do enjoy eating blueberries and
- 3 students who don't enjoy eating apples and don't enjoy eating blueberries.

We can calculate probabilities from Venn diagrams. Recall that a **probability** is calculated using

$$P(\text{event}) = \frac{\text{the number of favourable outcomes}}{\text{the total number of outcomes}}$$

Example



Question: Calculate the following probabilities using the information in the Venn diagram on the right. Give fractional answers in simplified form.
i. P(A)
ii. P(A or B or both)

Thought process: The number of outcomes for A is 9+13=22. The total number of outcomes is 9+13+8+4=34. So P(A)=22/34=11/17. The number of outcomes for A or B or both is 9+13+8=30. So P(A or B or both)=30/34=15/17

Answer: i. P(A)=11/17 ii. P(A or B or both)=15/17

29 statistics

Questions Part 1 of 9 – Ordering numbers

29.1 Order the following sets of numbers from smallest to largest.

a. 4, 7, 9, 0, 3, 7, 4   b. 8, 1, 2, 6, 1   c. 3, 5, 3, 9, 3

d. 9, 1, 7, 0, 2, 5   e. 2, 6, 9, 2, 2, 1, 4, 1, 4   f. 4, 1, 5, 0, 8, 7

g. 2, 6, 2, 0, 4   h. 6, 6, 1, 0, 9   i. 3, 1, 0, 8, 3, 5

Answers

a. 0, 3, 4, 4, 7, 7, 9 b. 1, 1, 2, 6, 8 c. 3, 3, 3, 5, 9 d. 0, 1, 2, 5, 7, 9 e. 1, 1, 2, 2, 2, 4, 4, 6, 9 f. 0, 1, 4, 5, 7, 8 g. 0, 2, 2, 4, 6 h. 0, 1, 6, 6, 9 i. 0, 1, 3, 3, 5, 8

The median is the middle value of an ordered set of data. The first sub-skill for finding the median is being able to order data from smallest to largest.

Example

Question: Order the following set of numbers from smallest to largest

4, 7, 9, 0, 3, 7, 4

Thought process: The smallest number in the set is 0, the 3, 4, 4, 7, 7, 9. To check that we haven't missed any numbers we count the numbers in the original list, 7, and count the numbers in our ordered list – 7 too!

Answer: 0, 3, 4, 4, 7, 7, 9

Questions Part 2 of 9 – Finding the number halfway between two numbers

29.2 What number is exactly halfway between each pair of numbers?

a. 2 and 9	b. 2 and 4	c. 3 and 8
d. 2 and 2	e. 2 and 3	f. 3 and 6
g. 4 and 6	h. 5 and 8	i. 2 and 8
j. 3 and 5	k. 1 and 6	l. 4 and 4

Answers

a. 5.5 b. 3 c. 5.5 d. 2 e. 2.5 f. 4.5 g. 5 h. 6.5 i. 5 j. 4 k. 3.5 l. 4

Helpful Information

Sometimes the halfway point is obvious, for example halfway between 2 and 3 is 2.5 and halfway between 13 and 15 is 14. Though sometimes it is a bit harder, for example finding halfway between 7 and 15 or halfway between 2 and 11 isn't so easy.

Below gives two strategies for finding the number halfway between more difficult pairs.

Strategy 1 for finding the halfway point between two numbers

1. Find the difference between the numbers
2. Halve this difference
3. Add half this difference to the starting number
4. Check that this number is the same distance from the final number.

Strategy 2 for finding the halfway point between two numbers

1. Calculate the mean of the two numbers.

The advantage of using Strategy 1 is that it is quite intuitive (it makes good sense) where the advantage of using Strategy 2 is that it is quite simple to remember.

Examples

Question: Calculate the number halfway between 2 and 9

Thought process: Following Strategy 1 above we have

1. $9 - 2 = 7$
2. $7 \div 2 = 3.5$
3. $2 + 3.5 = 5.5$
4. $5.5 + 3.5 = 9$

Answer: 5.5

Question: Calculate the number halfway between 3 and 12

Thought process: Following Strategy 2 above we calculate the mean of 3 and 12. First we calculate the sum, 3+12=15. Then we divide the sum by the number of numbers, 15÷2=7.5

Answer: 7.5

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Questions Part 3 of 9 – Calculating the median – odd amount of numbers

29.3 Calculate the median for the following sets of numbers.

a. 3, 7, 2, 0, 7	b. 5, 9, 2, 2, 7, 9, 6	c. 4, 8, 3, 4, 1, 1, 0
d. 7, 6, 8, 9, 6, 0, 0, 1, 1	e. 3, 0, 1, 1, 7	f. 6, 7, 7, 4, 1, 3, 9
g. 1, 3, 5, 0, 1	h. 1, 4, 1, 1, 9, 4, 5	i. 2, 5, 3, 3, 6, 7, 6

Answers

a. 3 b. 6 c. 3 d. 6 e. 1 f. 6 g. 1 h. 4 i. 5

Helpful Information

The **median** is the middle value of the ordered data.

Example

Question: Calculate the median for the following set of numbers

3, 7, 2, 0, 7

Thought process: We first order the data

0, 2, <u>3</u>, 7, 7

The median will be the middle number which is 3.

Answer: 3

Questions Part 4 of 9 – Calculating the median – even amount of numbers

29.4 Calculate the median for the following sets of numbers.

a. 9, 2, 7, 0, 2, 5	b. 4, 9, 3, 8, 11, 1	c. 6, 7, 1, 0, 3, 2, 3, 6
d. 7, 1, 2, 2	e. 3, 0, 3, 7, 8, 9	f. 2, 9, 12, 3, 8, 6, 7, 1
g. 4, 4, 2, 3, 1, 6	h. 2, 0, 9, 8, 9, 2	i. 3, 3, 1, 0, 1, 4, 0, 5

Answers

a. 3.5 b. 6 c. 3 d. 2 e. 5 f. 6.5 g. 3.5 h. 5 i. 2

Helpful Information

The **median** is the middle value of the ordered data. If there is an even number of data points the median is the number exactly half way between the two middle numbers.

Example

Question: Calculate the median for the following set of numbers

9, 2, 7, 0, 2, 5

Thought process: We first order the data

0, 2, <u>2, 5</u>, 7, 9

The median will be halfway between the two middle numbers. The difference between 2 and 5 is 3 so the halfway mark is at 2+1.5=3.5 (check 3.5+1.5=5).

Answer: 3.5

Questions Part 5 of 9 – Calculating the median – any amount of numbers

29.5 Calculate the median for the following sets of numbers.

a. 2, 6, 8, 0, 5	b. 6, 9, 3, 7, 11, 6	c. 3, 8, 9, 0, 6, 1, 3
d. 5, 8, 7, 6, 5, 0, 1, 1	e. 4, 0, 5, 0, 8, 7	f. 8, 1, 4, 5, 1, 0, 0
g. 5, 1, 1, 6, 0, 8, 6, 3	h. 1, 7, 1, 4, 5	i. 6, 9, 7, 8, 0
j. 9, 6, 9, 8, 3	k. 8, 1, 0, 8, 4, 5	l. 7, 2, 1, 0, 9

Answers

a. 5 b. 6.5 c. 3 d. 5 e. 4.5 f. 1 g. 4 h. 4 i. 7 j. 8 k. 4.5 l. 2

Helpful Information

The **median** is the middle value of the ordered data. If there is an even number of data points the median is the number exactly half way between the two middle numbers.

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Questions Part 6 of 9 – Calculating the mean

29.6 Calculate the mean for the following sets of numbers. Give the mean to one decimal place where appropriate.

a. 9, 2, 7, 0, 2, 5	b. 6, 9, 3, 7, 11, 6	c. 3, 8, 9, 0, 6, 1, 3
d. 5, 8, 7, 6, 5, 0, 1, 1	e. 4, 0, 5, 0, 8, 7	f. 8, 1, 4, 5, 1, 0, 0
g. 5, 1, 1, 6, 0, 8, 6, 3	h. 1, 7, 1, 4, 5	i. 6, 9, 7, 8, 0
j. 9, 6, 9, 8, 3	k. 8, 1, 0, 8, 4, 5	l. 7, 2, 1, 0, 9

Answers

a. 4.2 b. 7 c. 4.3 d. 4.1 e. 4 f. 2.7 g. 3.8 h. 3.6 i. 6 j. 7 k. 4.3 l. 3.8

Helpful Information

The **sum** of a group of numbers is found by adding the numbers together.

The **mean** is the sum of all the data points divided by the number of data points.

Example

Question: Calculate the mean for the following set of numbers. Give the mean to one decimal place where appropriate.

9, 2, 7, 0, 2, 5

Thought process: We calculate the sum of the data points: 9+2+7+0+2+5=25. There are 6 data points and 25÷6=4.166.. When rounded to one decimal place this is 4.2

Answer: 4.2

Questions Part 7 of 9 – Calculating the mode

29.7 Calculate the mode for the data sets below.

a. 2, 6, 8, 2, 5	b. 5, 9, 3, 7, 11, 6	c. 3, 8, 9, 0, 6, 6, 3
d. 5, 8, 7, 6, 5, 4, 1, 1	e. 8, 1, 3, 16	f. 8, 1, 4, 5, 1, 0, 0
g. 5, 1, 1, 6, 0, 8, 6, 3	h. 1, 7, 1, 4, 5	i. 6, 9, 7, 8, 0
j. 9, 6, 9, 8, 3	k. 8, 1, 0, 8, 4, 5	l. 7, 2, 1, 0, 9
m. 4, 9, 3, 8, 11, 1	n. 5, 9, 2, 2, 7, 9, 6	o. 2, 9, 12, 3, 8, 6, 7, 1

Answers

a. 2 b. no mode c. 3 and 6 d. 1 and 5 e. no mode f. 0 and 1 g. 1 and 6 h. 1 i. no mode j. 9 k. 8 l. no mode m. no mode n. 2 and 9 o. no mode

Helpful Information

The **mode** is the value (or values) which occur the most.

Examples

Question: Calculate the mode for the data set 2, 6, 8, 2, 5.

Thought process: The mode is the value which occurs the most. Ordering the data gives us 2, 2, 5, 6, 8. We can now easily see that 2 occurs more than any other value.

Answer: 2

Question: Calculate the mode for the data set 5, 9, 3, 7, 11, 6.

Thought process: The mode is the value which occurs the most. Ordering the data gives us 3, 5, 6, 7, 9, 11. We can now easily see that no number occurs more than any other.

Answer: No mode

Question: Calculate the mode for the data set 3, 8, 9, 0, 6, 6, 3.

Thought process: The mode is the value which occurs the most. Ordering the data gives us 0, 3, 3, 6, 6, 8, 9. We can now easily see that 3 and 6 occur more than any other values.

Answer: 3 and 6

Questions Part 8 of 9 – Calculating the mode **29.8** Calculate the range for the data sets below. a. 2, 6, 8, 2, 5 b. 5, 9, 3, 7, 11, 6 c. 3, 8, 9, 0, 6, 6, 3 d. 5, 8, 7, 6, 5, 4, 1, 1 e. 8, 1, 3, 16 f. 8, 1, 4, 5, 1, 0, 0 g. 5, 1, 1, 6, 0, 8, 6, 3 h. 1, 7, 1, 4, 5 i. 6, 9, 7, 8, 0 j. 9, 6, 9, 8, 3 k. 8, 1, 0, 8, 4, 5 l. 7, 2, 1, 0, 9 m. 4, 9, 3, 8, 11, 1 n. 5, 9, 2, 2, 7, 9, 6 o. 2, 9, 12, 3, 8, 6, 7, 1 *Answers* a. 6 b. 8 c. 9 d. 7 e. 15 f. 8 g. 8 h. 6 i. 9 j. 6 k. 8 l. 9 m. 10 n. 7 o. 11	*Helpful Information* The **range** is the highest value take away the lowest value. *Examples* **Question:** Calculate the range for the data set 2, 6, 8, 2, 5. **Thought process:**. Ordering the data gives us 2, 2, 5, 6, 8. The biggest value is 8 and the smallest value is 2. The range is equal to 8 - 2 = 6. **Answer:** 6 **Question:** Calculate the range for the data set 5, 9, 3, 7, 11, 6. **Thought process:** Ordering the data gives us 3, 5, 6, 7, 9, 11. The biggest value is 11 and the smallest value is 3. The range is equal to 11 - 3 = 8. **Answer:** 8 **Question:** Calculate the range for the data set 3, 8, 9, 0, 6, 6, 3. **Thought process:** Ordering the data gives us 0, 3, 3, 6, 6, 8, 9. The biggest value is 9 and the smallest value is 0. The range is equal to 9 - 0 = 9. **Answer:** 9
Questions Part 9 of 9 – Remembering the key statistical words **29.9** Re-write the amazing song on the right in your workbook and celebrate learning the final skill within this book!! *Answers* Hey diddle diddle, the **median's** the *middle, you add and divide* for the **mean, t**he **mode** is the one that you see the *most,* and the **range** is the *difference between!*	*Helpful Information* The summary statistics can be remembered using the following song: ♫ Hey diddle diddle The **median's** the *middle* You *add and divide* for the **mean** The **mode** is the one that you see the *most* And the **range** is the *difference between* ♫